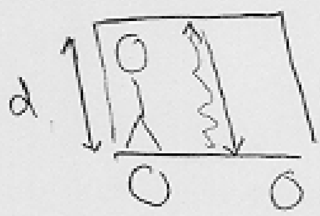
Time Dilation

- The speed of light is constant in all frames

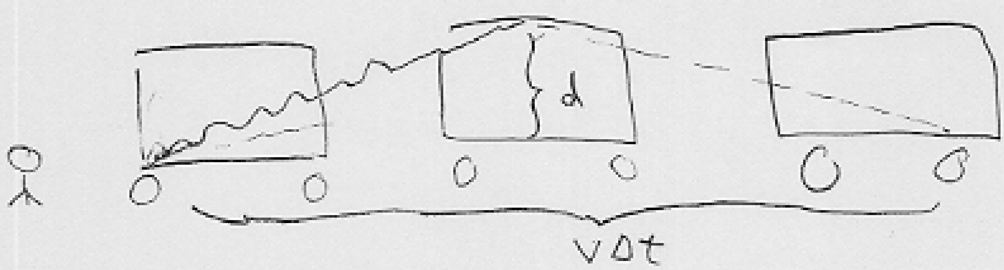
Time it take for person sitting still to throw and catch the light



$$\Delta\tau = \frac{2d}{c}$$

↑ proper time: time measured at the same position at spac

For an observer moving to the left with speed v



$$\frac{\text{Total Distance}}{\text{Total time}} = \text{speed of Light} = c$$

$$\frac{2\sqrt{d^2 + (v\Delta t/2)^2}}{\Delta t} = c \qquad \text{Solver For } \Delta t:$$

$$\sqrt{(2d)^2 + (v\Delta t)^2} = c\,\Delta t$$

$$(2d)^2 + (v\Delta t)^2 = c^2\Delta t^2$$

Skip this algebra, after saying solver for Dt jump to boxed formula.

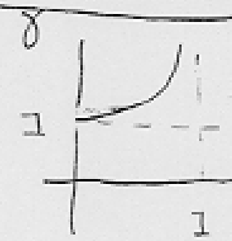
$$\left(\frac{2d}{c}\right)^2 + \left(\frac{v}{c}\right)^2 \Delta t^2 = \Delta t^2$$

$$\left(\frac{2d}{c}\right)^2 = \left[1 - \left(\frac{v}{c}\right)^2\right](\Delta t)^2$$

$$\frac{\left(\frac{2d}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2\right)^2} = (\Delta t)^2 \qquad \Delta t$$

$$\boxed{\Delta t = \left(\frac{2d}{c}\right) \frac{1}{\sqrt{1-(v/c)^2}} \implies \Delta t = \frac{\tau}{\sqrt{1-(v/c)^2}}}$$

Define $\boxed{\gamma = \frac{1}{\sqrt{1-(v/c)^2}}}$



- **Moving clocks run slow**

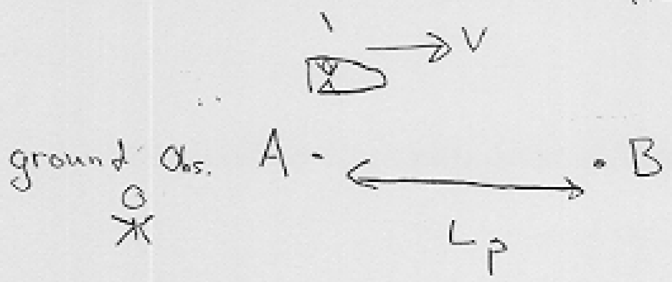
$$\Delta t = \gamma \Delta \tau \qquad \gamma > 1$$

time for someone on ground

time interval for someone on train

Length Contraction

Consider a Ruler Stick:



proper length = length as measured by someone at rest w.r.t. the ruler stick

Consider the ground observer he sees that the space-ship takes a time to complete his journey

$$\Delta t = \frac{L_p}{v} \qquad \Delta \tau_s = \frac{L}{v}$$

Now

$$\gamma \Delta \tau = \frac{L_p}{v}$$

$$\Delta \tau = \frac{(L_p/\gamma)}{v} = \frac{\text{Length of ruler as seen by space g}}{v}$$

$$L = \frac{L_p}{\gamma}$$

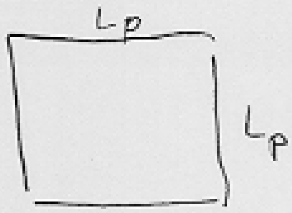
proper length

$\boxed{\text{Moving ruler sticks are length contracted}}$

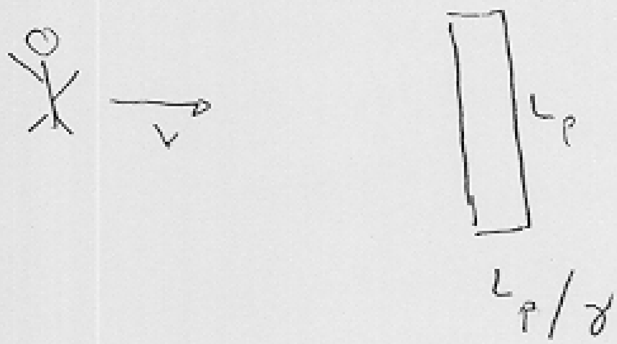
Remark:

- Only those directions in the direction of motion are Length contracted.
- Transverse Directions not contracted.

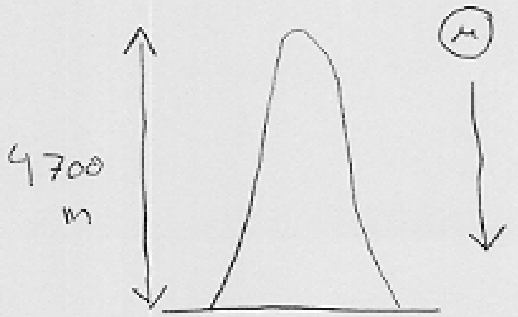
Fixed Observer:



Moving observer:



Muon and mountain : Earth Observer



$V_{muon} = 0.99c$

- The μ decays in $2.2\,\mu s$ in its own frame (a proper time)
- To an observer on earth the muon decays in

$$\Delta t = \gamma\, \Delta\tau \qquad \gamma = \frac{1}{\sqrt{1-(v/c)^2}} = 7.1$$

$$\Delta t = (7.1)(2.2\,\mu s) \simeq 16\,\mu s$$

- The distance travelled is $d = v\,\Delta t \approx c \cdot 16\,\mu s =$ 4700 meters
  the muon reaches the bottom!

Muon and mountain : Muon Observer

$$L = L_0/\gamma = \frac{4700\,m}{7.1} \simeq 65$$ 650 meters

The muon says $x = \Delta\tau\, v$ amount of mow passes him

$$x = 2.2\,\mu s\,(0.99c) \simeq 650\,m$$