Problem 1. Zangwill 14.24: Eddy current levitation

Problem 2. Zangwill 14.21: A rotating magnet.

Problem 3. Zangwill 14.23: Azimuthal eddy currents in a wire

Problem 4. Electric and Magnetic fields of AC Solenoid

A cylindrical solenoid of high conductivity and radius a carries surface current $\mathbf{K} = K_o \cos(\omega t)\hat{\boldsymbol{\phi}}$

(a) Determine the electric and magnetic fields to the first non-vanishing order in the quasi-static approximation.

(b) Show that the magnetic field to the next-to-leading order in the quasi-static approximation outside the cylinder is

$$\Delta B = \delta B_z(\rho) - \delta B_z(\rho_{\rm max}) = \frac{K_o}{c} \cos(\omega t)\, \tfrac{1}{2}(\omega a/c)^2 \left(- \log \frac{\rho}{a} + C \right) \tag{1}$$

where C $= \log \rho_{\rm max}/a$. Here we are quoting ΔB the difference between δB at ρ and δB at $\rho_{\rm max}$.

(c) The cutoff $\rho_{\rm max}$ arises because the quasi static approximation breaks down for large ρ where the physics of radiation becomes important. $\rho_{\rm max}$ should be of order $\rho_{\rm max} \sim c/\omega$. Explain qualitatively why the approximation breaks down for this radius.

Remark: Certainly $|\delta B_z(\rho_{\rm max})|$ is logarithmically smaller than $|\delta B_z(\rho)|$ for $\rho \sim a$. In a logarithmic approximation we can neglect $\delta B_z(\rho_{\rm max})$ and set $\rho_{\rm max} = c/\omega$ leading to

$$\delta B_z(\rho) \simeq \frac{K_o}{c} \cos(\omega t)\, \tfrac{1}{2}(\omega a/c)^2 \left(- \log \frac{\rho}{a} + \log(c/(\omega a)) \right) \tag{2}$$

Leading log accuracy may not be familiar to you. It just says that we are neglecting the constant inside the logarithm which is of order 1. Thus in this approximation,

$$\log(100/2) = \log(100) - \log(2) \simeq \log(100) \tag{3}$$

$$3.9 \simeq 4.6 \tag{4}$$

which is often good enough for government work. Bethe famously used such approximations to estimate the first QED corrections to the hydrodgen spectrum.

(d) Determine the magnetic field (to the next-to-leading order in the quasi-static approximation) inside the cylinder to logarithmic accuracy, and qualitatively sketch the complete magnetic field $B(\rho)/B_o$ where B_o is the leading order answer in the center of the cylinder.

Remark: Note that the ρ depence of part (b) and part (c) does not depend on the value of $C = \log(\rho_{\rm max}/a)$.

(e) Determine the vector and scalar potentials in the Coulomb and Lorentz gauges to the required order and accuracy to reproduce the electric and magnetic fields in part (a) and verify that you obtain the correct fields.