Last Time

(1) • Magneto Statics in matter:

$$\nabla \times B = \vec{j}/c \qquad \text{for steady or slow currents} \quad \nabla \cdot \vec{j} = 0$$

$$\nabla \cdot B = 0$$

(2) • Then the presence of a magnetic field in the material generates currents

$\frac{j_{mat}}{c}$ = Expansion in magnetic fields and its derivatives

weak field approximation

$$\frac{j_{mat}}{c} = \vec{\nabla} \times \vec{M}(B) \simeq \chi_m^B \, \vec{\nabla} \times \vec{B}$$

• first term in derivative expansion consistent $\nabla \cdot \vec{j} = 0$

• All other therms suppressed by powers of $\frac{l_{micro}}{L}$

Then

$$\nabla \times B = \nabla \times M + \frac{j_{free}}{c}$$

$$\nabla \times \underbrace{(B - M)}_{\equiv H} = \frac{j_{free}}{c}$$

Then

$$\boxed{\begin{aligned}\nabla\times H &= \vec{j}_{fr}/c\\ \nabla\cdot B &= 0\end{aligned}}\Bigg\} \Rightarrow$$ Usually work with H so $M(H)$ rather than $M(B)$.

For a linear material,

$$B(H) \equiv H + M(H) = \mu H = \frac{1}{(1-\chi_m^B)} H \equiv (1+\chi_m) H$$

We will use these equations today

(3) Then we estimated the size on χ_m^B

$$\frac{j_{mat}}{c} \sim \chi_m^B \, \vec{\nabla}\times B$$

- naive dimensions says $\chi_m \sim \frac{v_{mat}}{c}$

  since $j_{mat} \sim q v_{mat}$

- But ultimately, the medium currents caused by B arise from magnetic forces, $F_B = q \frac{v_{mat}}{c} \vec{B}$

So

$$j_{mat} \sim q v_{mat} \left(\frac{v_{mat}}{c}\right) B$$

• And thus $\qquad \beta = \alpha = \frac{1}{137}$

$$\chi_m \sim \left(\frac{v_{mat}}{c}\right)^2 \sim \left(\frac{1}{137}\right)^2 \sim 10^{-5}$$

For paramagnetic and diamagnet

Classification of Magnetic Materials

- **Diamagnetic** (oppose ≡ dia)

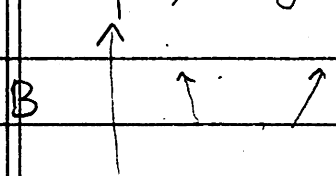
$\chi_m^B < 0$ and $\mu < 1$ $\chi_m^B \sim 10^{-5}$

Typically this is related to the orbital motion of the electrons, with all spins are paired. The orbits change to oppose the change in flux

- **Paramagnetism** (same ≡ para)

Typically this is related to the spin, aligning itself in mag field.

$\chi_m^B \sim 10^{-5}$



$\chi_m^B > 0$ $\mu > 1$

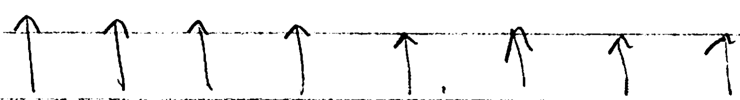
Ferromagnetism (Your Kitchen magnet inspired a lot of physics)

$$B = \mu(H)\ H$$

This number can be very large.

$\mu(H) \sim 10^3$.

The reason this number can be large is it involves a strong cooperation between all of the atoms, leading to a large magnetic moment per volume. The spins align



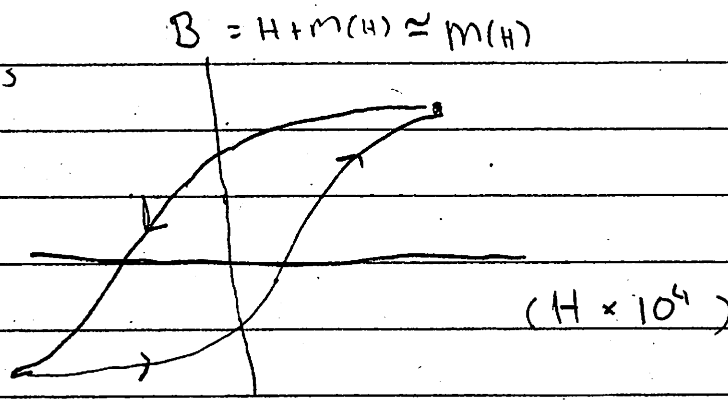
- Why do spins align? The magnetic field from one diopole would cause the neighboring spin to flip.

- The reason why the neighboring spins ~~tend~~ tend to align is electrostatic. When the spins are aligned spin wave-fcn can be symmetric, allowing the spatial wave-fcn to be anti-symmetric minimizing the Coulomb interaction energy. This is a <u>much</u> larger effect (by $\left(\frac{v}{c}\right)^2$)

than any magnetic effect.

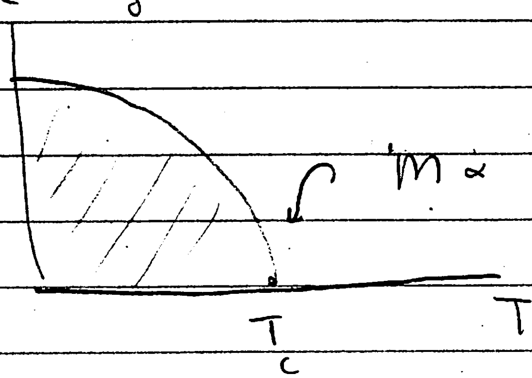
Phenomenology of Ferromagnets

(1) Hysersis



(2) Permanent Magnets:

$M \leftarrow$ magnetization in absence of field



$M \propto |T - T_c|^{\beta}$

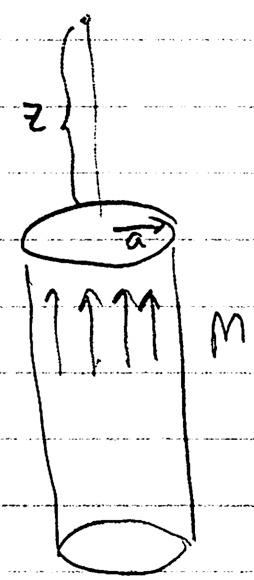
$\beta = 0.34$ for Fe

$T_c \sim 700^\circ C$

T_c ← critical point

__Example__ Problem with a hard ferro-magnet

- A uniformly magnetized cylinder, of height h, and Magnetization $\vec{M} = M_0 \hat{z}$
- two methods vector potential + scalar potential
  homeork
- Determine the magnetic field along the z-axis

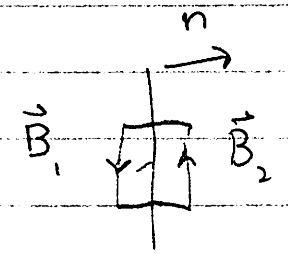


- $\vec{J} = \vec{\nabla} \times \vec{M} = 0$ inside:

But we have boundary conditions which give us a surface current

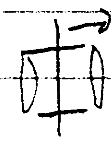
Previously analyzed (☆☆)

$$\nabla \times B = j/c$$



$$n \times (\vec{B}_2 - \vec{B}_1) = \frac{K_{TOT}}{c}$$

$$\nabla \cdot B = 0$$



$$n \cdot (\vec{B}_2 - \vec{B}_1) = 0$$

In matter

$$\nabla \times H = j_{fr}/c \quad \Longrightarrow \quad \boxed{\begin{array}{c} n \times (\vec{H}_2 - \vec{H}_1) = \frac{K_{free}}{c} \\ n \cdot (\vec{B}_2 - \vec{B}_1) = 0 \end{array}} \quad (☆)$$

$$\nabla \cdot B = 0$$

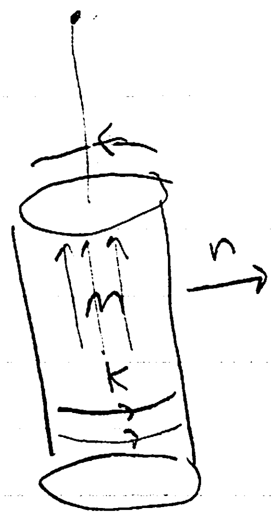
Using

$H = B - M$ and $j_{TOT} = j_{mat} + j_{free}$ in ☆ & ☆☆

Find

$$\boxed{n \times (\vec{m}_2 - \vec{m}_1) = \vec{K}_{mat}/c}$$

So



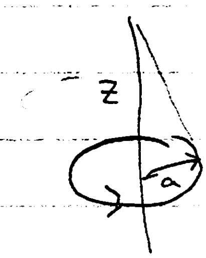
$$-\hat{n}\times M_0\hat{z} = \vec{K}_{mat}/c$$

$$M_0\hat{z}\times n = \vec{K}_{mat}/c$$

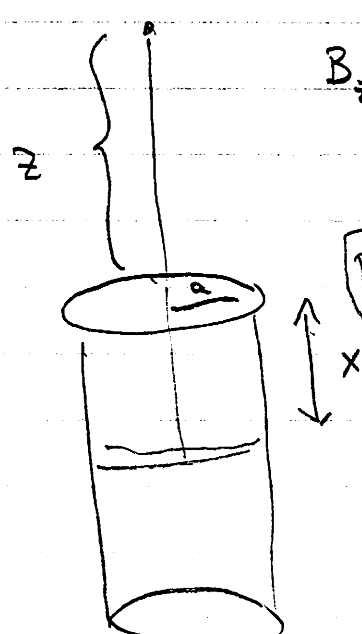
$$M_0\hat{\phi} = K_{mat}/c$$

So.... we find a cylindrically symmetric current.
Using, the field from a ring

$$dB = \frac{I}{c}\,\frac{a^2/2}{(a^2+z^2)^{3/2}}$$



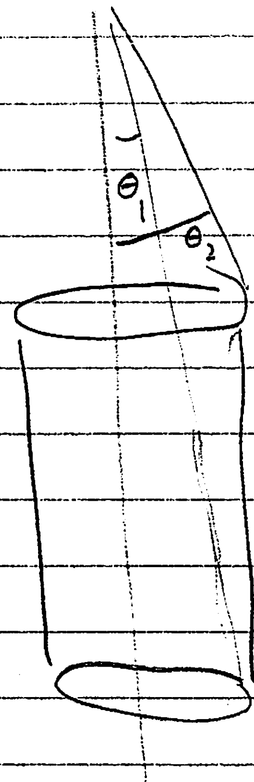
Then one can integrate



$$B_z = \int_0^h dx\, M\,\frac{a^2/2}{((z+x)^2+a^2)^{3/2}}$$

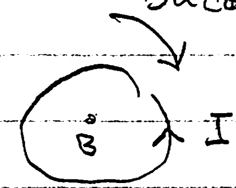
$$\boxed{B_z = \frac{M}{2}\left[\frac{(h+z)}{\sqrt{a^2+(h+z)^2}} - \frac{z}{\sqrt{a^2+z^2}}\right]}$$

Picture



$$B_z = \frac{m}{2}\left[\cos\Theta_1 - \cos\Theta_2\right]$$

Energy Stored in Magnetic Fields - Faraday's Law



Back emf or E-field opposes the increase in current

- Imagine slowly turning on the current. The increasing magnetic field (i.e changing) creates a back emf. We will need to fight the emf doing work on the system. The amount of work done in bringing the current up to full strength is the energy stored in magnetic field.

Now Lets look at the math

$$\nabla\cdot E = \cancel{\rho}^{\,0}$$ (since $\nabla\cdot\vec{j}\simeq 0$ for slowly changing fields

$$\nabla\times H = j/c + \cancel{\partial_t \vec{E}}$$ ← E field is small and changing slowly

$$\nabla\cdot B = 0$$

minus sign means the induced emf opposes change.

$$\nabla\times E = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$ ⟸ Changing $\vec{B}$ makes $\vec{E}$

Now $\nabla \cdot \vec{B} = 0$ So we can always introduce,

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

But $\nabla \times E \neq 0$ so can't write $E = -\nabla \varphi$.

Looking at

$$\nabla \times E = -\frac{1}{c}\frac{\partial}{\partial t} B$$

$$\nabla \times E = -\frac{1}{c}\frac{\partial}{\partial t} \nabla \times \vec{A}$$

So, $\nabla \times \left(\vec{E} + \frac{1}{c}\frac{\partial \vec{A}}{\partial t} \right) = 0$. Thus in general

can write

$$\boxed{\vec{E} + \frac{1}{c}\frac{\partial \vec{A}}{\partial t} = -\nabla \varphi}$$

And

$$\boxed{E = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \nabla \varphi}$$