Plane Waves in simple matter

$\nabla \cdot D = 0$

$\nabla \times H = \frac{1}{c}\partial_t D$

$\nabla \cdot B = 0$

$\nabla \times E = -\frac{1}{c}\partial_t B$

First limit to Harmonic Fields $e^{-i\omega t}$

$\nabla \times H = -\frac{i\omega}{c} D$ writing $D = \varepsilon E$

$B = \mu H$

$\nabla \times E = +\frac{i\omega}{c} \vec{B}$

Then find

$$\nabla \times \frac{(\nabla \times H)}{\frac{-i\omega\varepsilon}{c}} = +\frac{i\omega\mu}{c} H$$

Becomes:

The Helmholtz eqn

$$\boxed{\left(\nabla^2 + \frac{\omega^2\mu\varepsilon}{c^2}\right) H = 0}$$

Similary

$$\boxed{\left[\nabla^2 + \omega^2\left(\frac{\mu\varepsilon}{n}\right)\right] E = 0}$$

So we try a solution :

$$E = \vec{\mathcal{E}} e^{i\vec{k}\cdot\vec{x} - i\omega t}$$

$$B = \vec{\mathcal{B}} e^{i\vec{k}\cdot\vec{x} - i\omega t}$$

Find

$$k^2 + \omega^2\left(\frac{\mu\varepsilon}{c^2}\right) = 0 \Rightarrow \omega = \frac{ck}{n}$$

- Where $\boxed{n = \sqrt{\mu\varepsilon}}$ is the index of refraction
- The phase velocity of the waves is

$$v_\phi = \frac{\omega}{k} = \frac{c}{n}$$

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Find

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- Where $\boxed{n = \sqrt{\mu\varepsilon}}$ is the index of refraction
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Now the Divergence Eqs:

$$\left.\begin{array}{l} \nabla \cdot E = 0 \\ \nabla \cdot B = 0 \end{array}\right\}$$

give for our trial solutions

$$\left.\begin{array}{l} \vec{k} \cdot \vec{\mathcal{E}} = 0 \\ \vec{k} \cdot \vec{\mathcal{B}} = 0 \end{array}\right\}$$

Thus the vectors $\mathcal{E}$ & $\mathcal{B}$ are transverse to the beam

Finnally we have

$$\nabla \times E = -\frac{1}{c} \partial_t B$$

$$i\vec{k} \times \vec{\mathcal{E}} = i \frac{\omega}{c} \vec{\mathcal{B}}$$

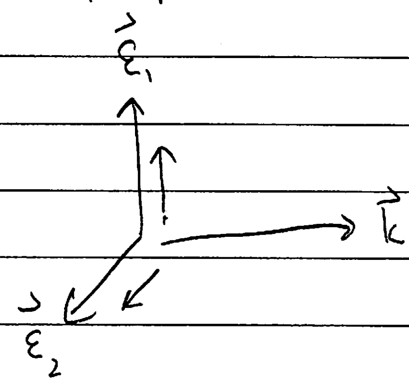
For $|k| = \frac{\omega}{(c/n)}$ find

$$\boxed{\sqrt{\mu \varepsilon}\, \frac{\vec{k}}{k} \times \vec{\mathcal{E}} = \vec{\mathcal{B}}}$$

← E & B are orthogonal

→ $\mathcal{B}$ is multiplied $n = \sqrt{\mu \varepsilon}$ relative to $\mathcal{E}$

Then construct two vectors which are perpendicular to $\vec{k}$:



$$\vec{\mathcal{E}} = \vec{\varepsilon}_1 E_0 \qquad \mathcal{B} = \sqrt{\mu\varepsilon}\, \hat{k} \times \mathcal{E}$$

$$= \vec{\varepsilon}_2 \sqrt{\mu\varepsilon}\, E_0$$

So we can use these to parametrize $\vec{E}$ and $\vec{B}$,

or

$$\vec{\mathcal{E}} = \vec{\varepsilon}_2 E_0 \qquad \mathcal{B} = \sqrt{\mu\varepsilon}\, \hat{k} \times \mathcal{E}$$

$$\vec{\mathcal{B}} = -\sqrt{\mu\varepsilon}\, \vec{\varepsilon}_1$$

Now the time averaged Poynting vector

$$\vec{S} = \left\langle c \left(\text{Re}\, \vec{E} e^{-i\omega t}\right) \times \text{Re}\left(\vec{H}\, e^{-i\omega t}\right) \right\rangle_{\text{Time}}$$

$$= \left\langle c \left[\frac{\vec{E} e^{-i\omega t} + \vec{E}^* e^{i\omega t}}{2}\right] \times \left[\frac{\vec{H} e^{-i\omega t} + \vec{H}^* e^{i\omega t}}{2}\right] \right\rangle_{\text{ti}}$$

$$= \frac{1}{4} \vec{c} \quad \vec{E} \times H^* + E^* \times \vec{H} +$$

$$\boxed{\vec{S}_{av} = \frac{1}{2} \text{Re} \quad c\vec{E} \times \vec{H}^*}$$

So then:

$$S = \frac{1}{2} c \frac{E_0 B_0}{\mu} = \frac{1}{2} c \sqrt{\frac{\varepsilon}{\mu}} |E_0|^2$$

Similarly:

$$\langle u \rangle = \frac{1}{4}\left(\varepsilon E \cdot E^* + \frac{B \cdot B^*}{\mu}\right)$$

$$\boxed{\langle u \rangle = \frac{1}{2} \varepsilon |E_0|^2}$$

Polarization

- In general

$$\vec{E} = (\vec{\varepsilon}_1 E_1 + \vec{\varepsilon}_2 E_2)\, e^{i\vec{k}\cdot x - \omega t}$$

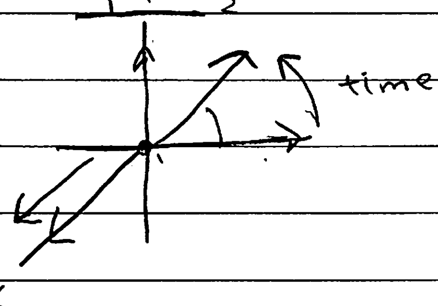
→ If E_1 and E_2 are in phase the light is linearly polarized

→ If E_1 and E_2 are 90° out of phase the light is circulary polarized

$$\vec{E} = E_0 \overbrace{(\vec{\varepsilon}_1 \pm i \vec{\varepsilon}_2)}^{\equiv\, \varepsilon_\pm} e^{i\vec{k}\cdot x - \omega t}$$

$$\mathrm{Re}\,\vec{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} E_0 \cos(kz - \omega t) \\ \mp E_0 \sin(kz - \omega t) \end{pmatrix}$$

$\underline{\varepsilon_1 + i\varepsilon_2}$



$$= \begin{pmatrix} E_0 \cos(\omega t - kz) \\ \pm E_0 \sin(\omega t - kz) \end{pmatrix}$$

So $\varepsilon_1 + i\varepsilon_2$ has <u>positive</u> helicity (

and $\varepsilon_1 - i\varepsilon_2$ has <u>negative</u> helicity

In general characterize the light

$$\vec{E} = (E_+ \vec{\varepsilon}_+ + E_- \vec{\varepsilon}_-)\, e^{i\vec{k}\vec{x} - \omega t}$$

By its density matrix:

$$\vec{E}\vec{E}^* \Longrightarrow \rho \equiv \begin{pmatrix} |E_+|^2 & E_- E_+^* \\ E_+ E_-^* & |E_-|^2 \end{pmatrix}$$

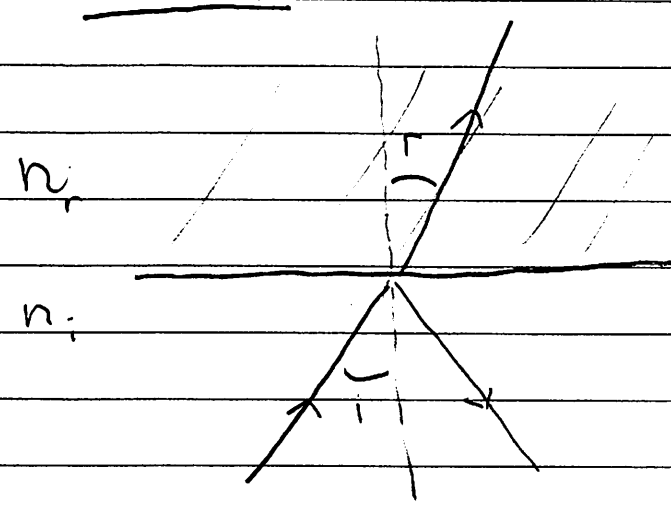
Then you can write

$$\rho = S_0 \cdot \frac{\sigma_0}{2} + S_i \frac{\sigma_i}{2} \Longrightarrow \begin{array}{l} S_0 = \mathrm{Tr}[\sigma_0 \rho] \\ S_i = \mathrm{Tr}[\sigma_i \rho] \end{array}$$

Where S_0 and S_i are known in optics as the Stokes parameters.

Reflection of Light at Interfaces

Qualitative

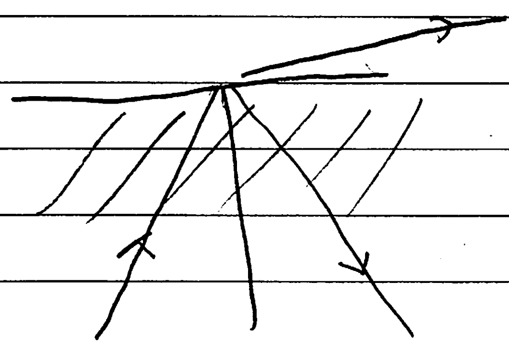


i = incidence angle.

r = refracted angle

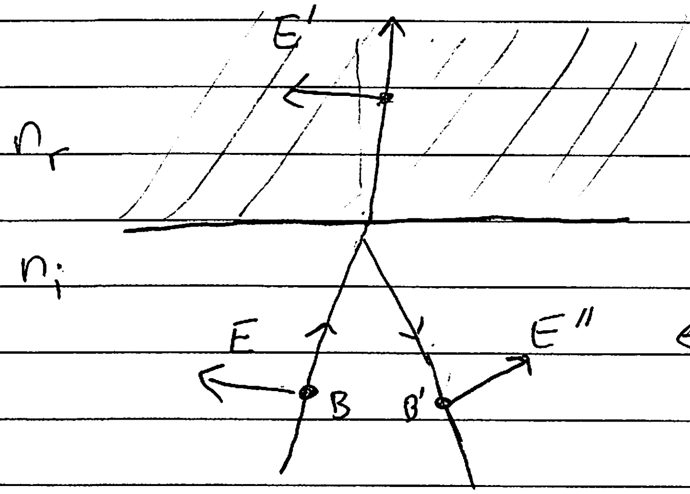
$$\frac{\sin i}{\sin r} = \frac{n_r}{n_i} \Rightarrow$$ for $n_r > n_i$ $\sin\theta_r < \sin\theta_i$

But for $n_r < n_i$ $\sin\theta_r > \sin\theta_i$



For $\sin\theta_r > 90^\circ$ get total internal reflection

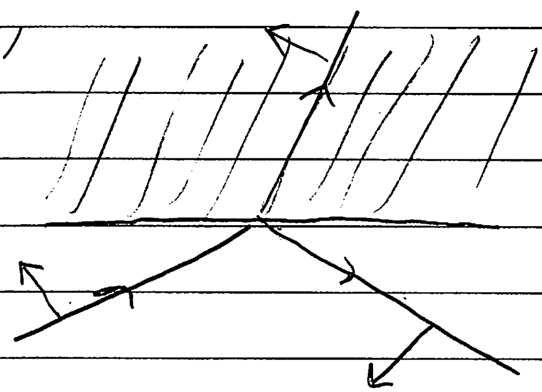
In Plane Polarization



Small angle picture

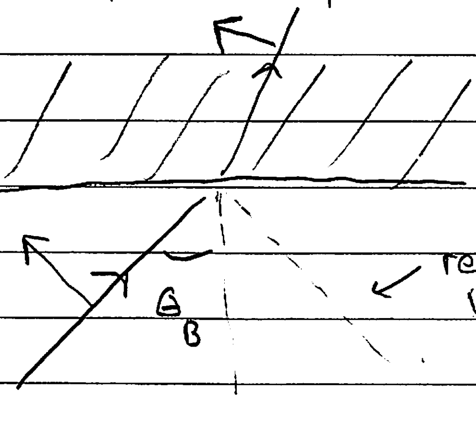
← Polarization inverted upon reflection

But,



For small angles the polarization is not inverted

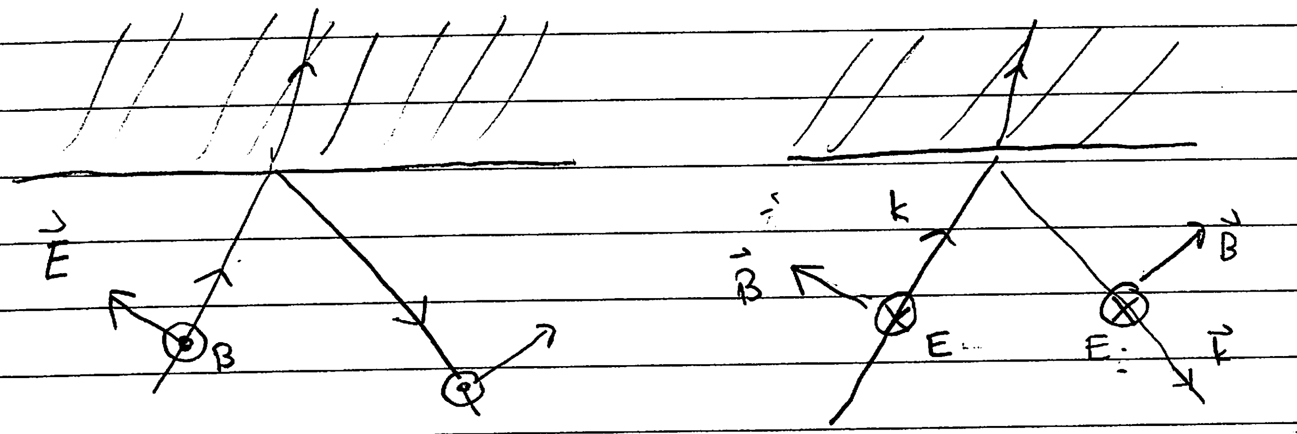
So there must exsist an angle were the in plane polarization is <u>not</u> reflected.



← reflected wave Vanisches

$$\tan \Theta_B = \frac{n_r}{n_i}$$ is the Brewster angle.

In general the incident light is polarized in plane and out of plane



in plane polarization

perp to plane polarization

- After going through the interface the reflected / transmitted light will be polarized out of plane / in plane.

- This is used by radio towers to select the transmitted radio waves