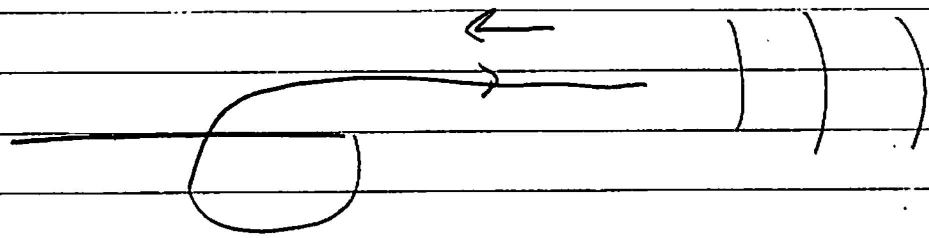
Radiation from Relativistic Charges



$$-\Box\varphi = \rho$$
$$-\Box\vec{A} = \vec{J}/c$$

$$G(tr|t_0 r_0) = \frac{1}{4\pi|\vec{r}-\vec{r}_0|}\,\delta(t - t_0 - |\vec{r}-\vec{r}_0|/c)$$

Then from the Grn-fcn for the wave eqn

$$\varphi(t,\vec{r}) = \int dt_0\, d^3r_0\; \delta\left(t - t_0 - \frac{|\vec{r}-\vec{r}_0|}{c}\right) \frac{\rho(t_0 r_0)}{4\pi|\vec{r}-\vec{r}_0|}$$

$$\vec{A}(t,\vec{r}) = \int dt_0\, d^3r_0\; \delta\left(t - t_0 - \frac{|\vec{r}-\vec{r}_0|}{c}\right) \frac{\vec{J}/c\,(t_0, r_0)}{4\pi|\vec{r}-\vec{r}_0|}$$

For a point-charge

$$\rho(t_0, r_0) = e\,\delta^3(r_0 - r_*(t_0))$$
$$\vec{J}(t_0, r_0) = e\,v(t_0)\,\delta^3(\vec{r}_0 - \vec{r}_*(t_0))$$

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We now do the $d^3 r_0$ integrals

$$\varphi(t,\vec{r}) = \int dt_0 \ \delta\left(t - t_0 - \frac{|\vec{r} - r_*(t_0)|}{c}\right) \frac{e}{4\pi R}$$

$$\vec{A}(t,\vec{r}) = \int dt_0 \ \delta\left(t - t_0 - \frac{|\vec{r} - r_*(t_0)|}{c}\right) \frac{e V(t_0)}{4\pi R}$$

Now we do the integral

- Only one value of t_0 will contribute $T(t,r)$ determined implicitly

$$\delta\left(t - t_0 - \frac{|\vec{r} - \vec{r}_*(t_0)|}{c}\right)$$

$$= \delta\left(t_0 - t + \frac{|\vec{r} - r_*(t_0)|}{c}\right)$$

Using

$$\delta(f(t_0)) = \frac{\delta(t_0 - T)}{|f'(T)|}$$

with

$$\frac{d}{dt_0}\left(t_0 - t + \frac{|\vec{r} - r_*(t_0)|}{c}\right) = 1 + \frac{1}{c}\frac{d}{dt}\left((r - r_*(t))^2\right)^{1/2}$$

$$= 1 - \frac{\vec{n} \cdot V_*(t_0)}{c} \qquad V_*(t_0) = \frac{dr_*(t_0)}{dt_0}$$

$$\vec{n} \equiv \frac{\vec{r} - \vec{r}_*(t)}{|\vec{r} - r_*(t_0)|}$$

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So we find

$$\boxed{\begin{aligned} \varphi(t,r) &= \frac{e}{4\pi R(T)} \frac{1}{\left(1 - n\cdot \frac{v(T)}{c}\right)} \\ \vec{A}(t,\vec{r}) &= \frac{e\vec{v}(T)}{4\pi R(T)} \frac{1}{\left(1 - n\cdot \frac{v}{c}(T)\right)} \end{aligned}}$$

These

$$T \equiv t - \frac{|r - r_*(T)|}{c}$$ ← determines T implicitly as function of t, r

$$R = |\vec{r} - \vec{r}_*(T)|$$

$$\frac{\partial T}{\partial t} = 1 + n\cdot v \frac{\partial T}{\partial t}$$

$$\frac{\partial T}{\partial t} = \frac{1}{(1 - n\cdot v)}$$