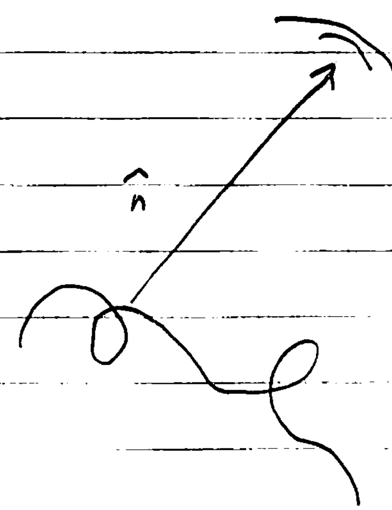
Last Times

$$-\Box \vec{A} = \vec{J}/c$$

Solving and taking the far field limit



$$\vec{A}_{rad} = \frac{e}{4\pi r} \frac{\vec{V}(T)/c}{(1 - n\cdot\beta)}$$

Then we compute

$$\vec{E}_{rad} = \frac{e}{4\pi r}\, \vec{n}\times\vec{n}\times \frac{1}{c}\frac{\partial \vec{A}_{rad}}{\partial t}$$

use $\frac{\partial T}{\partial t} = \frac{1}{(1-n\cdot\beta)}$

$$= \frac{e}{4\pi r}\, \vec{n}\times\vec{n}\times \frac{1}{(1-n\cdot\beta)} \frac{1}{c}\frac{\partial A}{\partial T}$$

Then

$$E_{rad} = \frac{e}{4\pi r c^2} \overbrace{\frac{1}{(1-n\cdot\beta)}}^{dT/dt} \cdot \frac{d}{dT} \frac{n\times n\times V}{(1-n\cdot\beta)}$$

$$= \frac{e}{4\pi r c^2} \frac{1}{(1-n\cdot\beta)} \frac{n\times(n-\beta)\times a}{(1-n\cdot\beta)^2}$$

$$T \equiv t - \frac{r}{c} + \frac{n\cdot r_*(T)}{c} \quad \text{in the far field}$$

Then we also computed the frequency spectrum

$$E_{rad}(\omega, r) = \int_{-\infty}^{\infty} e^{i\omega t}\, dt \;\; E_{rad}(t, r)$$

Using the fact that $dT/dt = 1/(1 - n\cdot\beta)$ find, changing integral from $dt \to dT$

$$E_{rad}(\omega, r) = \frac{q}{4\pi r c^2} e^{i\omega r/c} \int_{-\infty}^{\infty} dT\, e^{i\omega(T - n\cdot r_*(T))} \frac{d}{dT} \frac{\hat{n}\times\vec{n}\times\vec{v}}{(1 - n\cdot\beta)}$$

Or

$$E_{rad}(\omega, r) = \frac{q\,(-i\omega)}{4\pi r c^2} e^{i\omega r/c} \int_{-\infty}^{\infty} dT \;\; e^{i\omega(T - n\cdot r_*(T))} \; n\times n\times v(T) \; dT$$

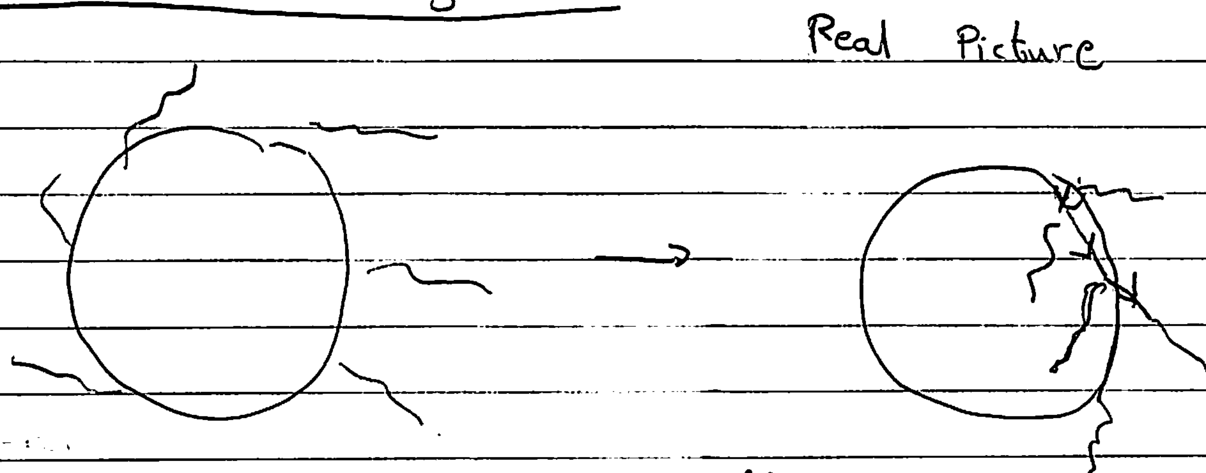
The energy spectrum

$$2\pi \frac{dW}{d\omega\, d\Omega} = c\, |r\, E_{rad}(\omega, r)|^2$$

Ultimate the classical radiation spectrum describes the distribution of photons

$$2\pi \frac{dW}{d\omega\, d\Omega} = 2\pi \hbar\omega \frac{dN}{d\omega\, d\Omega}$$

Limitation of Electro-dynamics



Treated the radiation as not affecting the trajectory

$$\frac{\hbar\omega}{c} \ll \gamma m v$$

Also treated the trajectory classically

$$\lambda_D \sim \frac{\hbar}{p} \ll R \quad \leftarrow \text{So as } \gamma \to \infty \text{ this condition is increasingly well satified}$$

So with, $\omega_{typ} \sim \frac{\gamma^3 c}{R_0}$, then

$$\hbar \cdot \frac{\gamma^3 c}{R_0} \ll \gamma m \overset{c}{\tilde{v}}$$

So

$$\gamma^2 \ll \frac{R_o}{\hbar/mc}$$

Or

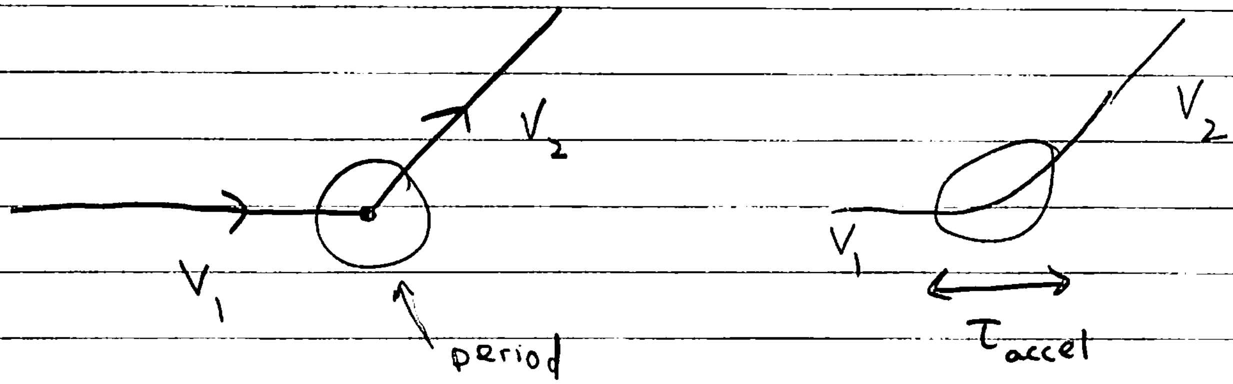
$$\gamma \ll \sqrt{\frac{R_o}{\hbar/mc}} \sim \sqrt{\frac{100\,m}{10^{-12}\,m}} \sim 10^7$$

or $E_e \sim 10$ TeV

electron compton wavelength

Bremsstrahlung

Now lets compute the distribution of photons during a collision



period of rapid acceleration at $T=0$ $r_*(0)=0$

Then to compute the fourier spectrum we return to

$$E_{rad}(\omega) = \frac{q\, e^{i\omega r/c}}{4\pi r c^2}\,(-i\omega)\int_{-\infty}^{\infty} dT\, e^{i\omega(T - r_*(T)\cdot \frac{n}{c})}\; n\times n\times v(T)$$

Easier to use

$$E_{rad}(\omega) = \frac{q\, e^{i\omega r/c}}{4\pi r c^2}\int_{-\infty}^{\infty} dT\, e^{i\omega(T - r_*\cdot n/c)}\,\frac{d}{dT}\,\frac{\vec{n}\times n\times \vec{v}(T)}{(1-n\cdot\beta)}$$

Over a short time the phase is approximately one since $T=0$ $r_*(0)=0$ (or constant). At late times $\vec{a}$ is zero. so we can easily integrate around $T\simeq 0$

$$E_{rad}(\omega) \simeq \frac{q}{4\pi r c^2} e^{i\phi} \left[\frac{n \times n \times v_2}{(1 - n \cdot \beta)} - \frac{n \times n \times v_1}{(1 - n \cdot \beta)} \right]$$

And

$$2\pi \frac{dW}{d\omega\, d\Omega} = \frac{q^2}{16\pi^2 c^3} \left| \frac{n \times n \times \vec{v}_2}{(1 - n \cdot \beta_2)} - \frac{n \times n \times v_1}{(1 - n \cdot \beta_1)} \right|^2$$

The polorized radiation spectrum can also be writteno down:

$$\vec{E}_{rad} = E_{\parallel} \vec{\varepsilon}_{\parallel} + E_{\perp} \vec{\varepsilon}_{\perp}$$

Or

$$2\pi \frac{dW}{d\omega\, d\Omega} = \frac{q^2}{16\pi^2 c^3} \left| \varepsilon \cdot E \right|^2$$

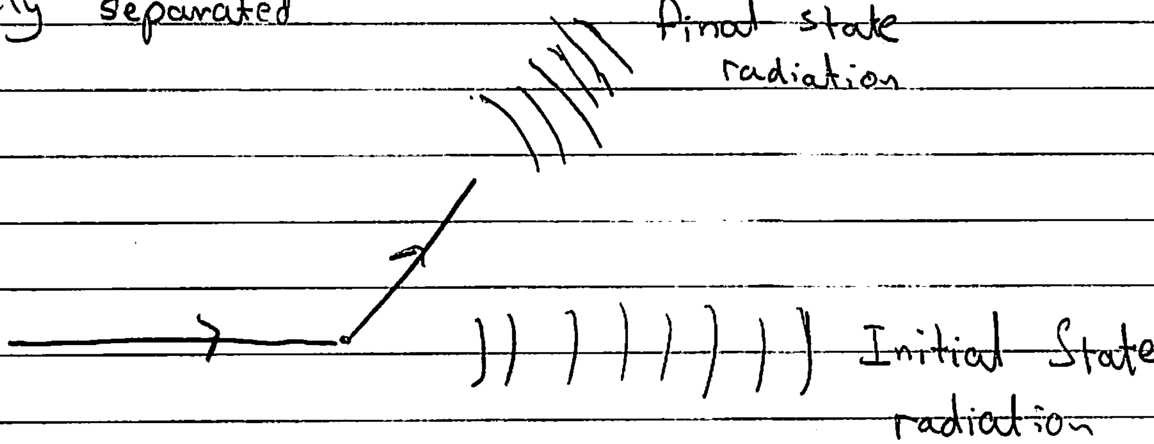
$$= \frac{q^2}{16\pi c^3} \left| \frac{\varepsilon \cdot v_2}{(1 - n \cdot \beta_2)} - \frac{\varepsilon \cdot v_1}{(1 - n \cdot \beta_1)} \right|^2$$

Now hets look at the qualitative features:

(1) There are two collinear factors

$$\frac{1}{(1-n\cdot V_2/c)} \quad \text{and} \quad \frac{1}{(1-n\cdot V_1/c)}$$

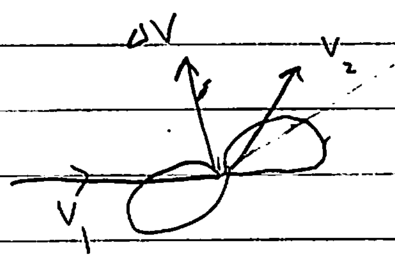
As long as V_1 and V_2 are separated by a wide angle, the radiation will be widely separated



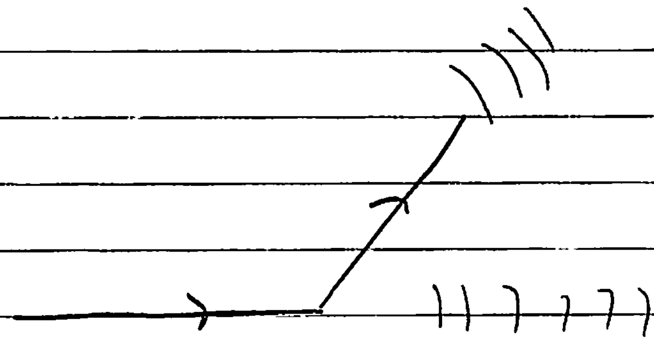
(2) In the non-relativistic limit

$$\frac{2\pi\, dW}{d\omega\, d\Omega} = \frac{q^2}{16\pi^2 c^3} \left| n\times n\times (\vec{v}_2-\vec{v}_1)\right|$$

Kind of Larmor Like.



③ Independent of frequency



Now

$$2\pi \frac{dW}{d\omega\, d\Omega} \simeq \omega \frac{dN_\gamma}{d\omega\, d\Omega} \cdot 2\pi$$

So

$$2\pi \frac{dN_\gamma}{d\Omega} = \frac{1}{\omega}\left(\frac{q^2}{16\pi^2 c^3}\right)\left| \frac{n\times n\times v_2}{(1-n\cdot\beta_2)} - \frac{n\times n\times v_1}{(1-\beta_1\cdot n)} \right|^2$$

characteristic infrared, $\frac{1}{\omega}$, yield of soft photons. The yield is infinite, but the energy they carry is finite

④ Now it also means that the energy is formally infinite

$$2\pi \frac{dW}{d\Omega} = \int_{-0}^{\Lambda} 2\pi \frac{dW}{d\omega\, d\Omega} \simeq \Lambda \left(\frac{q^2}{16\pi^2 c^3}\right)\left|\ldots\right|^2$$

The distribution of bremmed photons will be cutoff when either (A) formation time becomes comparable to the acceleration

$$E_{rad} \propto \int_{-\infty}^{\infty} e^{\overbrace{i\omega(T - \vec{r}_*(T)\cdot\frac{\vec{n}}{c})}^{\text{phase}}} \frac{d}{dT} \frac{(n \times n \times \vec{v}(T))}{(1 - n\cdot v(T))}$$

We neglected the change in phase over the duration of the collision

$$\Delta\phi \simeq \omega\left(\Delta T - \frac{n}{c}\cdot\frac{\Delta r_*}{\Delta T}\Delta T\right)$$

$$\Delta\phi = \omega\left(1 - \frac{n}{c}\cdot\vec{v}\right)\Delta T$$

So if the ΔT is of order the accel $\Delta T \sim \tau_{acc}$ then we can ignore the phase until

$$\omega\left(1 - \frac{n}{c}\cdot\vec{v}\right)\tau_{accel} \sim 1$$

$$\tau_{accel} \sim \frac{1}{\omega\left(1 - \frac{n}{c}\cdot v\right)}$$

this is what we called the formation time of the radiati of frequency ω

$$\tau_{accel} \sim \tau_{form}(\omega)$$

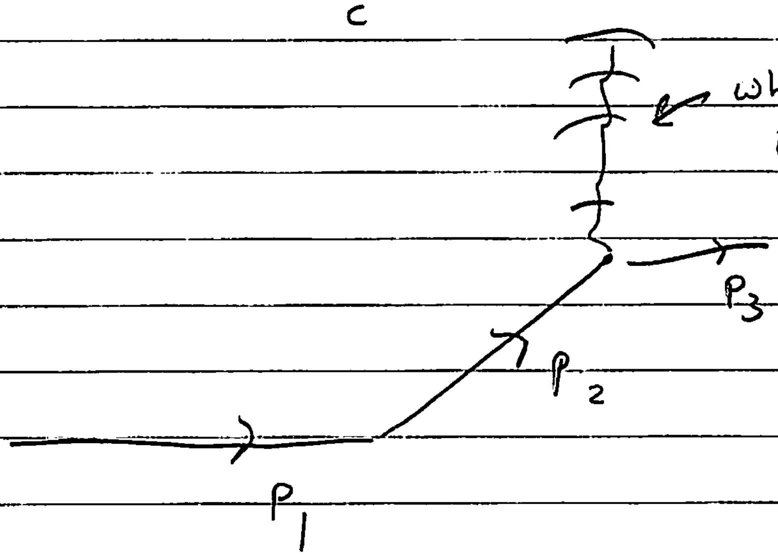
So

$$\omega_{max} \sim \frac{1}{\tau_{accel}\left(1 - \frac{n\cdot v}{c}\right)}$$

$$\omega_{max} \sim \frac{2\gamma^2}{\tau_{accel}\left(1+(\gamma\theta)^2\right)^2} \xrightarrow{\gamma\to\infty} \frac{1}{\tau_{accel}\,\theta^2}$$

(B) Another thing whicho can cut off the Bremsstrahlung spectrum is the finite energy of particles + quantum mechanics

$$\frac{\hbar\omega}{c} \ll p_1 \text{ and } p_2$$



when this photons momentum becomes comparable to p_2 can no longer treat the emission classically