Problem 1. Soft bremsstrahlung during a decay

In a collision or decay that happens at location $\boldsymbol{r}_o$ over an infinitessimally short time scale, $\tau_{\rm accel}$, the charged particles moving with velocity, $\mathbf{v}_1, \mathbf{v}_2, \ldots$ before the collsions and the charged particles moving with $\mathbf{v}_{1'}, \mathbf{v}_{2'}, \ldots$, after the collision each contribute to the radiation field. (The total radiation field is just a sum of the radiation fields from each particle.)

(a) Show that for frequencies low $\omega \ll 1/\tau_{\rm accel}$ the total radiation field is

$$\boldsymbol{E}_{\rm rad}(\omega, r) = e^{i\omega(r - \boldsymbol{n}\cdot\boldsymbol{r}_o)/c} \left(\sum_{j' \,\in\, \rm final} \frac{q_{j'}}{4\pi r c^2} \frac{\boldsymbol{n}\times\boldsymbol{n}\times\mathbf{v}_{j'}}{1 - \boldsymbol{n}\cdot\boldsymbol{\beta}_{j'}} - \sum_{j \,\in\, \rm initial} \frac{q_j}{4\pi r c^2} \frac{\boldsymbol{n}\times\boldsymbol{n}\times\mathbf{v}_j}{1 - \boldsymbol{n}\cdot\boldsymbol{\beta}_j} \right) \tag{1}$$

This generalizes the result of Lecture 46.

Hint. You may encounter an integral like

$$\int_0^\infty \boldsymbol{n}\times\boldsymbol{n}\times\mathbf{v}\; e^{i\omega T(1 - \boldsymbol{n}\cdot\mathbf{v}/c)} \,. \tag{2}$$

To give this integral definite meaning insert a convergence factor $e^{-\epsilon|T|}$ and then take the limit $\epsilon \to 0$ after integration. In any real experiment the velocity $\mathbf{v}(T)$ would be cut off in time, and provide this convergence factor naturally.

(b) A neutral ω^o meson of mass $M_\omega c^2 = 784\,\mathrm{MeV}$ has a relatively rare decay mode $\omega^o \to \pi^+\pi^-$, with branching fraction of 1.53%. (98.5% of the time it decays to something else.) It has another rare decay mode $\omega^o \to e^+e^-$ with branching ratio $7.28 \times 10^{-3}\%$. (These are pretty rare decays for the ω^o meson – most of the time it decays to $\pi^+\pi^-\pi^0$ with a branching fraction of 89.2%). The mass of a pion is $mc^2 = 140\,\mathrm{MeV}$, while the electron mass is ...

(i) Compute the frequency spectrum of the soft electromagnetic radiation per solid angle that accompanies both of these decay modes

$$\frac{dI}{d\omega d\Omega} = 2\left.\frac{dW}{d\omega d\Omega}\right|_{\omega>0} , \tag{3}$$

Describe your result qualitatively.

(ii) Show that for both of these decay modes the frequency spectrum of radiated energy at low frequencies is

$$\frac{dI}{d\omega} = \frac{e^2}{4\pi^2 c}\left[\left(\frac{1+\beta^2}{\beta}\right)\ln\frac{1+\beta}{1-\beta} - 2\right] \simeq \frac{e^2}{\pi^2 c}\left[\ln\left(\frac{M_\omega}{m}\right) - \frac{1}{2}\right] \tag{4}$$

where M_ω is the mass of the ω_o meson, m is the mass of one of the decay products, and β is the velocity/c of the decay products.

(iii) Roughly evaluate the total energy radiated in each decay by integrating the spectrum up to a point where the photon's momentum is half of the momentum of the decay products. (Beyond this point the recoil of the charged decay products would need to be considered. This lies outside of classical electrodynamics. In classical electrodynamics we specify the currents and solve for the fields.). You should find in a leading $\log(M_\omega/m)$ approximation

$$\frac{I_{\rm rough}}{M_\omega c^2} \simeq \frac{\alpha}{\pi} \log\left(\frac{M_\omega}{m}\right) \tag{5}$$

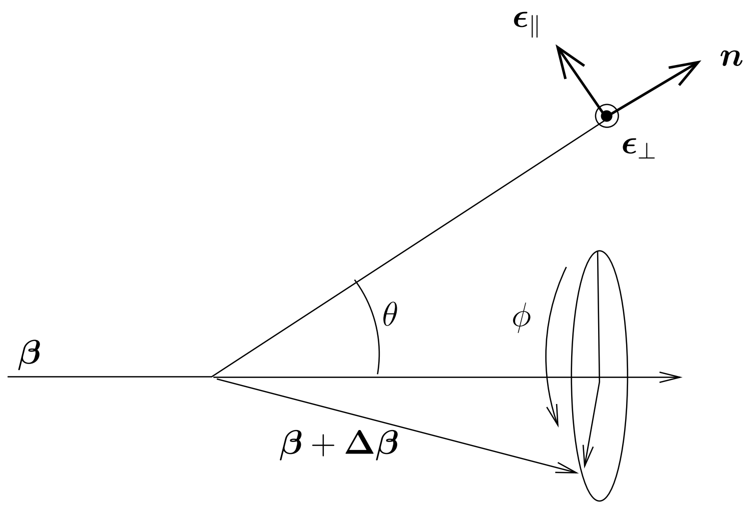
Using this rough evaluation, what fraction of the rest energy of the ω^o is carried away by soft radiation in the two decay modes

Problem 2. Polarization from random kick (based on Jackson 5.6)

(a) Show that for a grazing collision with a small velocity change $\boldsymbol{\Delta\beta}$, the intensity of the radiation with polarization $\boldsymbol{\epsilon}$ is (to first order in $\boldsymbol{\Delta\beta}$)

$$\frac{dI_\epsilon}{d\omega d\Omega} = \frac{q^2}{16\pi^2 c}\left|\boldsymbol{\epsilon}^* \cdot \left(\frac{\boldsymbol{\Delta\beta} + \boldsymbol{n} \times (\boldsymbol{\beta} \times \boldsymbol{\Delta\beta})}{(1 - \boldsymbol{n}\cdot\boldsymbol{\beta})^2}\right)\right|^2 \tag{6}$$

Below we will average over the azimuthal angle ϕ of $\boldsymbol{\Delta\beta}$.



(b) For the coordinates illustrated in the figure show that

$$\begin{aligned}
\boldsymbol{\epsilon}_\parallel \cdot [\boldsymbol{\Delta\beta} + \boldsymbol{n} \times (\boldsymbol{\beta} \times \boldsymbol{\Delta\beta})] =& |\boldsymbol{\Delta\beta}|(\beta - \cos\theta)\cos\phi && (7)\\
\boldsymbol{\epsilon}_\perp \cdot [\boldsymbol{\Delta\beta} + \boldsymbol{n} \times (\boldsymbol{\beta} \times \boldsymbol{\Delta\beta})] =& |\boldsymbol{\Delta\beta}|(1 - \beta\cos\theta)\sin\phi && (8)
\end{aligned}$$

(c) Use these results in the ultra-relativistic limit to show that that the distribtuion of photons with parallel and transverse polarizations are (after averaging over the azimuthal angle ϕ of the velocity change) is

$$\begin{aligned}
\frac{dI_\parallel}{d\omega d\Omega} =& \frac{q^2\gamma^4}{8\pi^2\ c}|\Delta\beta|^2\frac{(\gamma^2\theta^2 - 1)^2}{(1 + \gamma^2\theta^2)^4} && (9)\\
\frac{dI_\perp}{d\omega d\Omega} =& \frac{q^2\gamma^4}{8\pi^2 c}|\Delta\beta|^2\frac{1}{(1 + \gamma^2\theta^2)^2} && (10)
\end{aligned}$$

(d) Thus, show that for an electron experiencing a random transverse kick of magnitude $|\Delta\boldsymbol{p}_\perp|^2$ (which determines the velocity change), the radiated light at angle θ is partially polarized

$$P(\theta) \equiv \frac{\frac{dI_\perp}{d\omega d\Omega} - \frac{dI_\parallel}{d\omega d\Omega}}{\frac{dI_\perp}{d\omega d\Omega} + \frac{dI_\parallel}{d\omega d\Omega}} = \frac{2\gamma^2\theta^2}{1 + \gamma^4\theta^4} \tag{11}$$