Problem 1. (Optional. In class. Good exam material) Radiation during perpendicular acceleration

Consider an ultrarelativistic particle of velocity \( \beta \) experiencing an acceleration \( a_\perp \) perpendicular to the direction of motion. Here \( a_\perp \) points along the \( x \)-axis and \( \beta \) points along the \( z \)-axis.

(a) Show that the energy radiated per retarded time is approximately

\[
\frac{dW}{dT d\Omega} = \frac{e^2}{16\pi^2 c^3} \frac{a_\perp^2}{(1 - \beta \cos \theta)^3} \left[ 1 - \frac{\sin^2 \theta \cos^2 \phi}{\gamma^2(1 - \beta \cos \theta)^2} \right] \]

(1)

\[
\approx \frac{e^2}{2\pi^2 c^3} \frac{a_\perp^2}{(1 + (\gamma \theta)^2)^3} \left[ 1 - \frac{4(\gamma \theta)^2 \cos^2 \phi}{(1 + (\gamma \theta)^2)} \right] \]

(2)

In the first equality, I give the full answer without approximation, but I will only grade the second approximate result.

Hint, in working out this radiation pattern you might (as a start) show without approximation that

\[
|n \times (n - \beta) \times a|^2 = (1 - n \cdot \beta)^2 a^2 - (n \cdot a)^2 (1 - \beta^2) \]

(3)

by using the "b(ac)-(ab)c" rule. Then select a coordinate system were

\[
\beta = (0, 0, \beta) \]

(4)

\[
a = (a_\perp, 0, 0) \]

(5)

\[
n = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \]

(6)

(b) Work in a ultra-relativistic approximation, and compute the total power by integrating over the solid angle (as done in class) to show that you obtain the appropriate relativistic Larmour result\(^1\)

\[
\frac{dW}{dT} = \text{come on ... you know it ... right?} \]

(7)

\[^1\]

\[
\frac{dW}{dT} = \frac{e^2}{4\pi^2 \gamma^6} \frac{2}{3} a_\perp^2 \]

\[
\frac{dW}{dT} = \frac{e^2}{4\pi^2 \gamma^6} \frac{2}{3} \gamma^4 a_\perp^2 \]

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