Electrostatics in Media:

$$\nabla \cdot E = \rho$$

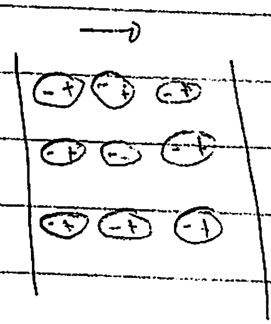
$$\nabla \times B = \frac{j}{c} + \frac{1}{c}\partial_t E \qquad \partial_t \rho + \nabla \cdot \vec{j} = 0$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{1}{c}\partial_t \vec{B}$$

Need to specify the currents in the medium in order to solve. Symmetry is key

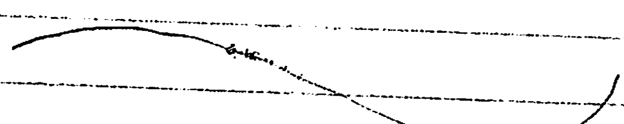
Basic Picture of Insulating Material



Electric field polarizes the insulating material

Key points:

① The wavelength of the external field is vastly longer than the microscopic scales



$\lambda_{light} \sim 500\ nm$

Where as size of atom $a_0 \sim \frac{1}{2} \mathring{A}$

$$\frac{\lambda_{light}}{a_0} \sim 10^4$$

(2) Fields are often weak compared to micro-fields

$$E_{ext} \sim 1 \frac{kV}{cm} \sim 10^5 \frac{V}{m} \quad \text{(Largest man-made field)}$$

$$E_{atom} \sim \frac{13.6\ \cancel{e}V}{\cancel{e}\ a_0} \sim 13 \times 10^{10} \frac{V}{m}$$

Now we will need these points and Symmetry

Parity

$t \rightarrow \underset{\sim}{t} = t$ (P-even)

$x \rightarrow \underset{\sim}{x} = -x$ (P-odd)

$\vec{p} \rightarrow \underset{\sim}{\vec{p}} = -\vec{p}$ (P-odd)

Prf:

$$\underset{\sim}{\vec{p}} = m \frac{d\underset{\sim}{x}}{d\underset{\sim}{t}} = -m \frac{dx}{dt} = -p$$

$F \rightarrow -\vec{F}$ (P-odd)

$\vec{L} \rightarrow \underset{\sim}{\vec{L}} = \vec{L}$ (P-even ≡ psuedo vector)

(Use $L = \vec{r} \times \vec{p}$)

$j \rightarrow \vec{\underset{\sim}{j}} = -j$ (P-odd)

$E \rightarrow -\vec{E}$ (P-odd)

$B \rightarrow B$ (P- even ≡ psuedo vector)

Use $F_E = q E$ and $F_B = q \vec{v} \times \vec{B}$

odd even odd odd even odd even

Time Reversal

$$t \longrightarrow \underset{\sim}{t} = -t \qquad \text{(T-odd)}$$

$$x \longrightarrow \underset{\sim}{x} = x \qquad \text{(T-even)}$$

$$\vec{p} \longrightarrow \underset{\sim}{p} = -p \qquad \text{(T-odd)}$$

Prf:

$$\underset{\sim}{p} = m \frac{d\underset{\sim}{x}}{d\underset{\sim}{t}} = -m \frac{dx}{dt} = -p$$

$$F \longrightarrow \underset{\sim}{F} = F \qquad \text{(T-even)}$$

(Use $\vec{F} = dp/dt$)

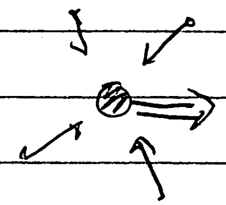
$$E \longrightarrow \underset{\sim}{E} = E \qquad \text{(T-even)}$$

(Use $F = qE$)

$$B \longrightarrow \underset{\sim}{B} = -B \qquad \text{(T-odd)}$$

(Use $\underset{\text{even}}{F} = q \underset{\text{odd}}{\vec{v}} \times \underset{\text{odd}}{\vec{B}}$)

Dissipation



Put a heavy particle interacting with light particles in a computer simulation

$$m\frac{d^2x}{dt^2} = \sum_i F_i \qquad\qquad F_i = \frac{q\,q_i}{4\pi|\vec{r}-r_i|}$$

The heavy partice experiences an <u>effective</u> force

$$M\frac{d^2x}{dt} = F_D$$

$$F_D = -\eta v$$

Now imagine running all particles in reverse $t \to \tilde{t} = -t$. The equations of motion are the same, and the heavy particle would still slow down. The effective force

$$\underset{\sim}{F_D} = -\underset{\sim}{\eta}\,\underset{\sim}{\vec{V}}$$

odd under t-reverse

even under time reversal

odd under t-reverse $\quad \tilde{\eta} = -\eta$

Summary

$$\eta \rightarrow \eta_{tr} = -\eta \quad \text{under time reversal}$$

Dissipative coëfficients are T-odd

Fourier Transforms:

$$\phi(t,\vec{r}) = \int \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i\omega t + i\vec{k}\cdot\vec{r}} \phi(\omega,k) \equiv (\phi(\omega,k))_{FT}$$

$$\phi(\omega,k) = \int dt\, d^3\vec{r}\ e^{i\omega t - i\vec{k}\cdot r} \phi(t,r) \equiv (\phi(t,r))$$

Slow & long wavelength means k, ω small

$$\phi(t,r) \longleftrightarrow \phi(\omega,k)$$

$$\partial_t \phi(t,r) \longleftrightarrow -i\omega\phi(\omega,k) = (\partial_t\phi)_{FT}$$

$$\cancel{\partial_i \phi(t,r) \longleftrightarrow i k_i \phi(\omega,k)}$$

Prf

$$\partial_t \phi(t,r) = \int \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} \partial_t (e^{-i\omega t + i\vec{k}\cdot\vec{r}}) \phi(\omega,k)$$

$$= \int \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i\omega t + ik\cdot r} - i\omega\phi(\omega,k)$$

Thus!

$$\vec{\nabla}\cdot\vec{E} = \rho(\vec{x})$$

$$(\nabla\cdot E)_{FT} = (\rho(x))_{FT}$$

$$(\partial_i E^i)_{FT} = (\rho(x))_{FT}$$

$$ik_i E^i(\omega,k) = \rho(\omega,k)$$