

A Heavside Lorenz (HL) Units

A.1 MKS to HL Units

- The HL Maxwell Equations follow from the MKS maxwell equations by defining

$$\mathbf{E}_{HL} = \sqrt{\epsilon_o} \mathbf{E}_{MKS} \qquad \mathbf{B}_{HL} = \frac{\mathbf{B}_{MKS}}{\sqrt{\mu_o}} \qquad (\text{A.1})$$

$$\rho_{HL} = \frac{\rho_{MKS}}{\sqrt{\epsilon}} \qquad \frac{\mathbf{j}_{HL}}{c} = \sqrt{\mu_o} \mathbf{j}_{MKS} \qquad (\text{A.2})$$

and using $c = 1/\sqrt{\epsilon_o \mu_o}$

- To convert from MKS to HL set $\epsilon_o = 1$ (and thus $\mu_o = 1/c^2$, $\sqrt{\mu_o} = 1/c$) and use this table

Quantity	$\epsilon_o = 1$ relation
B-field	$c\mathbf{B}_{MKS} = \mathbf{B}_{HL}$
A-field	$c\mathbf{A}_{MKS} = \mathbf{A}_{HL}$
magnetic dipole moment	$\frac{m_{MKS}}{c} = m_{HL}$
magnetization	$\frac{M_{MKS}}{c} = M_{HL}$
induction	$\frac{H_{MKS}}{c} = H_{HL}$
permeability	$\mu_{MKS}/\mu_o = \mu_{HL}$
permittivity	$\epsilon_{MKS}/\epsilon_o = \epsilon_{HL}$

Example: the magnetic potential energy

$$U_B = \frac{1}{2} \frac{B_{MKS}^2}{\mu_o} \Rightarrow \frac{1}{2} (cB_{MKS})^2 = \frac{1}{2} B_{HL}^2 \qquad (\text{A.3})$$

Example: The poynting vector

$$\mathbf{S} = \frac{1}{\mu_o} \mathbf{E}_{MKS} \times \mathbf{B}_{MKS} \Rightarrow c\mathbf{E}_{MKS} \times (c\mathbf{B}_{MKS}) = c\mathbf{E}_{HL} \times \mathbf{B}_{HL} \qquad (\text{A.4})$$

Example: The force law

$$\mathbf{F} = q_{MKS}(\mathbf{E}_{MKS} + \mathbf{v} \times \mathbf{B}_{MKS}) = q_{MKS}(\mathbf{E}_{MKS} + \frac{\mathbf{v}}{c} \times c\mathbf{B}_{MKS}) \Rightarrow q_{HL}(\mathbf{E}_{HL} + \frac{\mathbf{v}}{c} \times \mathbf{B}_{HL}) \qquad (\text{A.5})$$

Example: The magnetic energy of a dipole

$$U = -\mathbf{m}_{MKS} \cdot \mathbf{B}_{MKS} \Rightarrow -\frac{\mathbf{m}_{MKS}}{c} (c\mathbf{B}_{MKS}) = -\mathbf{m}_{HL} \cdot \mathbf{B}_{HL} \qquad (\text{A.6})$$

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Example: The Magnetic energy in matter

$$U = \frac{1}{2} \mathbf{B} \cdot \mathbf{H} \Rightarrow \frac{1}{2} c \mathbf{B}_{MKS} \frac{\mathbf{H}_{MKS}}{c} = \frac{1}{2} \mathbf{B}_{HL} \cdot \mathbf{H}_{HL} \quad (\text{A.8})$$

Example: Consistency of definition of \mathbf{H}

$$H = \frac{1}{\mu_o} B - M \Rightarrow H_{MKS} = c^2 B_{MKS} - M_{MKS} \Rightarrow H_{HL} = B_{HL} - M_{HL} \quad (\text{A.9})$$

The last step follows by dividing both sides by c .

A.2 HL to MKS

- The relation between charges and currents in the HL and MKS units are

$$Q_{HL} = \frac{Q_{MKS}}{\sqrt{\epsilon}} \quad \rightarrow \quad \frac{1}{\sqrt{\epsilon_o}} (1 \mu\text{C}) = 0.336 \sqrt{N \cdot m^2} \quad (\text{A.10})$$

$$\frac{I_{HL}}{c} = \frac{I_{MKS}}{\sqrt{\epsilon c}} = \sqrt{\mu_o} I \quad \rightarrow \quad \sqrt{\mu_o} (1 \text{ amp}) = 0.00112 \sqrt{N \cdot m^2} \quad (\text{A.11})$$

- The relation between Field strengths and is

$$E_{HL} = \sqrt{\epsilon_o} E_{MKS} \quad \rightarrow \quad \sqrt{\epsilon_o} (1 \text{ kV/cm}) = 0.2975 \sqrt{N/m^2} \quad (\text{A.12})$$

$$B_{HL} = \sqrt{\epsilon_o} (c B_{MKS}) = \frac{1}{\sqrt{\mu_o}} B_{MKS} \quad \rightarrow \quad \frac{1}{\sqrt{\mu_o}} (1 \text{ Tesla}) = 892.062 \sqrt{N/m^2} \quad (\text{A.13})$$