Model Dielectric Overview

We will present the Lorentz/Harmonic oscillator model for the dielectric response. Why?

1. The model captures certain qualitative features of dielectrics and metals.

   Later we will see this characteristic response is a consequence of causality.

2. The model gives simple and correct estimates for the size of medium parameters such as the susceptibility $\chi_0$ in insulators and $\sigma_0$ in metals.
Consider a model consisting of \( N \) electrons per volume harmonically bound to atomic centers in an external field \( E(t) = E_0 e^{-i\omega t} \).

Atomic electrons bound to protons

\[ E(t) = E_0 e^{-i\omega t} \]

Then the external field makes a current

\[ m \frac{d^2 x}{dt^2} + m\eta \frac{dx}{dt} + mw_0^2 x = E_0 e^{-i\omega t} \]

Here we have added a damping term which could represent collisions other atoms. The units are such that \( \eta \) represents the frequency of collisions. Below we define the collision time

\[ \gamma = \frac{1}{\tau_c} \]

Then solving Eq (1) we have \( x(t) = x_0 e^{-i\omega t} \)

\[ x_0 = \frac{eE_0/w}{[-w^2 + w_0^2 - i\omega\eta]} \]
The current \( \dot{J}(t) = e_n V(t) = j \omega e^{-i \omega t} \) is

\[
\dot{J} = \frac{ie^2/m}{[-\omega^2 + \omega_0^2 - i\omega\eta]} \quad (-i\omega E_0)
\]

So the susceptibility \( \chi_e(\omega) = \chi_e(\omega) (-i\omega E(\omega)) \) is

\[
\chi_e(\omega) = \frac{ie^2/m}{[-\omega^2 + \omega_0^2 - i\omega\eta]}
\]

Then writing in Real + i Imaginary parts

\[
\chi_e(\omega) = \frac{(ie^2/m)}{[\omega_0^2 - \omega^2]} + \frac{i (ne^2/m)}{[\omega^2 - \omega_0^2 + (\omega\eta)^2]}
\]

\[
\text{Re} \chi_e(\omega)
\]

\[
\text{Im} \chi_e(\omega)
\]

Thus we have \( \chi_e(\omega) = 1 + \chi_e(\omega) \)

\[
\text{Re} \chi_e(\omega)
\]

\[
\text{Im} \chi_e(\omega)
\]
This is often a reasonable model (see handout). Part of its success is because the model respects causality.
Comparison of the Lorentz Model to the dielectric response of silicon

Zangwill adapted from Wooten (1972)
Comparison with the low frequency limit

1. Insulator:

In the limit \( \omega \to 0 \), find

\[
\lim_{\omega \to 0} \chi_e(\omega) = \frac{ne^2}{\omega_p^2} = \frac{\omega_p^2}{\omega^2} \quad \omega_p^2 = \frac{ne^2}{m}
\]

The quantity \( \frac{ne^2}{m} \) is known as the (squared) plasma frequency. Let's estimate its value for typical solids and liquids:

\[
n \sim \frac{1}{a_o^3}, \quad \hbar \omega_0 \sim \frac{\hbar^2}{m a_o^2} \sim \frac{e^2}{a_o} \quad \text{(Think Bohr model)}
\]

Then, a typical atomic frequency

\[
\frac{\omega^2}{\omega_0} = \frac{ne^2}{m} \sim \frac{1}{m a_o^2} \sim \frac{1}{\hbar^2} \left( \frac{\hbar^2 e^2}{m a_o^2 a_o} \right) \sim \omega_0^2
\]

Thus, expect that the static susceptibility is of order unity.

\[
\chi_e = \frac{\omega_p^2}{\omega_0^2} \sim 1
\]

This is what is found:

- e.g. quartz \( \chi_e = 0.46 \)
- water \( \chi_e = 0.33 \)
Conductors:

Conductors have no binding frequency \( \omega_0 = 0 \). Define the conductivity by the low frequency limit

\[
\lim_{\omega \to 0} \chi_0(\omega) = i \frac{\sigma_0}{\omega}
\]

Comparison with the model

\[
\lim_{\omega \to 0} \chi_0(\omega) = i \frac{ne^2}{mp}
\]

So

\[
\sigma_0 = \omega_p^2 \tau_c \quad \text{with} \quad \omega_p^2 = \frac{ne^2}{m} \quad \tau_c = \frac{1}{\eta}
\]

\( \eta \) is the time between collisions with lattice defects and impurities.

Thus, the density of electrons in Cu determines the typical de Broglie wavelength and oscillation frequency \( \omega_p \sim 10^{15} \text{ Hz} \) of the conduction electrons. These conduction electrons form wave packets which behave quasi-classically occasionally scattering off lattice defects and impurities. For Copper \( \sigma_{\text{Cu}} \sim 10^{17} \text{ Hz} \), we find...
\[ \omega_p T_c \sim 100. \] Thus you should have the following quasi-classical picture of scattering of defects.

Electron wave packet flying for the conduction of electrons in metals.

\[ \text{Random Scatterings with defects} \]

\[ \text{quasi-particle} \]

This classical kinetic picture is well justified because the typical de Broglie wavelength is short compared to the spacings between collisions, \( \ell_{mf} \).

\[ k_{\text{De Broglie}} \ell_{mf} \gg 1 \]

Or since \( T_c \sim \ell_{mf} / V_f \) and \( \omega_p = k_{\text{De Broglie}} V_f / \)

\[ \omega_p T_c \gg 1. \]