14 Scattering

We formulated the scattering problem. In this case incoming light induces currents in the object, which in turn create a radiation field. We will work with small objects and weak scattering where the effect of the induced radiation fields can be neglected in determining the currents. The external incoming field will induce acceleration in the case of light-electron scattering, or induce time-dependent dipole moments (i.e. currents) in the case of light scattering off a sphere.

(a) The Electric field can be written

$$\boldsymbol{E} = \boldsymbol{E}_{\rm inc} + \boldsymbol{E}_{\rm scat} \tag{14.1}$$

where

$$\boldsymbol{E}_{\rm inc}(t, \boldsymbol{r}) = E_o\, \boldsymbol{\epsilon}_o e^{ikz - i\omega t} \tag{14.2}$$

while the scattered field falls of as $1/r$

$$\boldsymbol{E}_{\rm scat}(t, \boldsymbol{r}) \to \boldsymbol{C}(\boldsymbol{k}) \frac{e^{ikr - i\omega t}}{r} \tag{14.3}$$

$\boldsymbol{E}_{\rm scat}$ (in the far field) might as well be called $\boldsymbol{E}_{\rm rad}$. The constant is proportional for E_o for linear response and so the far field of the scattered field is written in terms of the scattering amplitude, $\boldsymbol{f}(\boldsymbol{k})$.

$$\boldsymbol{E}_{\rm scat}(t, \boldsymbol{r}) \to E_o \boldsymbol{f}(\boldsymbol{k}) \frac{e^{ikr - i\omega t}}{r} \tag{14.4}$$

(b) We will follow the following notation for harmonic fields. We write E_ω to notate the thing in front of $e^{-i\omega t}$

$$\boldsymbol{E}(t) = \boldsymbol{E}_\omega e^{-i\omega t} \tag{14.5}$$

Since writing $\boldsymbol{E}_{\omega,{\rm scat}}(\boldsymbol{r})$ gets old fast, we will just write $E_{\rm scat}(\boldsymbol{r})$ or simply $E_{\rm scat}$ without anything to mean $E_{\omega,{\rm scat}}(\boldsymbol{r})$ when clear from context

(c) The radiation field $\boldsymbol{E}_{\rm scat}$ can be decomposed into polarizations

$$\boldsymbol{E}_{\rm scat} = E_1 \boldsymbol{\epsilon}_1 + E_2 \boldsymbol{\epsilon}_2 \tag{14.6}$$

Using the orthogonality of the polarization vectors

$$\boldsymbol{\epsilon}_a^* \cdot \boldsymbol{\epsilon}_b = \delta_{ab}\,, \tag{14.7}$$

we have, *e.g.*

$$E_1 = \boldsymbol{\epsilon}_1^* \cdot \boldsymbol{E}_{\rm scat} \qquad E_2 = \boldsymbol{\epsilon}_2^* \cdot \boldsymbol{E}_{\rm scat}\,. \tag{14.8}$$

The time averaged power radiated per solid angle with polarization $\boldsymbol{\epsilon}_1$ is

$$\overline{\frac{dP}{d\Omega}}(\boldsymbol{\epsilon}_1; \boldsymbol{\epsilon}_o) = \frac{c}{2} |r\, \boldsymbol{\epsilon}_1^* \cdot \boldsymbol{E}_{\rm scat}|^2 \tag{14.9}$$

and similarly for $\boldsymbol{\epsilon}_2$. This will in general depend on the incoming polarization, $\boldsymbol{\epsilon}_o$, of the light.

(d) The cross section is the time averaged radiated power divided by the (time-averaged) input flux

$$\frac{d\sigma(\boldsymbol{\epsilon}; \boldsymbol{\epsilon}_o)}{d\Omega} = \frac{\overline{\frac{dP}{d\Omega}(\boldsymbol{\epsilon}_1; \boldsymbol{\epsilon}_o)}}{\frac{c}{2}|E_o|^2} = |\boldsymbol{\epsilon}_1^* \cdot \boldsymbol{f}(\boldsymbol{k})|^2 \tag{14.10}$$

Long Wavelength Scattering

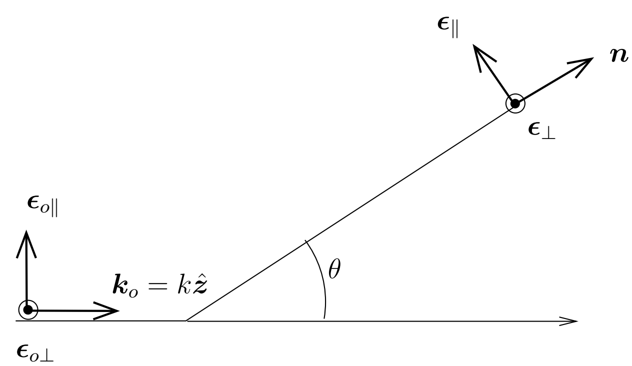
(a) We studied Thomson scattering (light-electron scatering) and found that the cross section was proportional to the classical electron radius squared

$$\sigma_T = \frac{8\pi}{3} r_e^2 \qquad r_e^2 = \left(\frac{q^2}{4\pi mc^2}\right)^2 \tag{14.11}$$

You should feel comfortable deriving this result and estimating the answer without looking up numbers. To derive the result compute the acceleration, and then compute the radiated electric field using Larmour type results (see Sect. (11.2) and Eq. (11.25) in particular). With the radiated field you can compute the power-radiated per solid angle with a given frequency.

(b) We also studied dipole scattering were we found that the cross section increases as ω^4. You should feel comfortable deriving this result. To derive the result you determine the induced dipole moment (electric, or magnetic, or both) in the applied field, and then use this induced dipole moment (which is oscillating) to compute the radiated field (see Eq. (11.33) and Eq. (11.42))

(c) The cross section for polarized scattering is found by considering the following picture:



So there are four cases depending on whether the incoming and outgoing polarizations are parallel or perpendicular to the scattering plane. For example, the cross section to produce light of polarization $\epsilon_{\perp}$ by un-polarized light (50% $\epsilon_{o\perp}$ and 50% $\epsilon_{o\parallel}$) is

$$\frac{d\sigma_{\perp}}{d\Omega} = \frac{1}{2}\left[\frac{d\sigma(\epsilon_{\perp};\epsilon_{o\perp})}{d\Omega} + \frac{d\sigma(\epsilon_{\perp};\epsilon_{o\parallel})}{d\Omega}\right] \tag{14.12}$$

Born Approximation

(a) We showed that the scattering amplitude and current can be expressed in terms of the induced current. The cross section to produce light of any polarization is the square of the scattering amplitude

$$\frac{d\sigma}{d\Omega} = |\boldsymbol{f}(\boldsymbol{k})|^2 = \frac{k^2}{16\pi^2 E_o^2}\left|\boldsymbol{n}\times\int d^3\boldsymbol{r}_o \frac{\boldsymbol{J}_\omega(\boldsymbol{r}_o)}{c} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_o}\right|^2 . \tag{14.13}$$

This is just a rewriting of Eq. (11.55) using the definitions used in scattering. In the scattering problem we must also determine the current.

(b) In a Born approximation, the current in a dielectric medium is determined only by the incoming electric field, since the scattered field is small

$$\boldsymbol{J}_\omega(\boldsymbol{r}) = -i\omega\chi(\omega,\boldsymbol{r})\,\boldsymbol{E}_{\omega,\mathrm{inc}}(\boldsymbol{r})\,, \tag{14.14}$$

where

$$\boldsymbol{E}_{\omega,\mathrm{inc}}(\boldsymbol{r}) = E_o\boldsymbol{\epsilon}_o e^{i\boldsymbol{k}_o\cdot\boldsymbol{r}_o} \qquad \text{with} \qquad \boldsymbol{k}_o \equiv k\hat{\mathbf{z}}\,. \tag{14.15}$$

So the cross section in this approximation is

$$\frac{d\sigma}{d\Omega} = \left(\frac{k^2}{4\pi}\right)^2 |\boldsymbol{n} \times \boldsymbol{\epsilon}_o|^2 \left| \int_V d^3\boldsymbol{r}_o \, \chi(\omega, \boldsymbol{r}_o) e^{i(\boldsymbol{k}-\boldsymbol{k}_o)\cdot\boldsymbol{r}_o} \right|^2 . \tag{14.16}$$