Reflection of Light at Interfaces - Introduction

\[ n_2 > 1 \]

\[ n_1 = 1 \]

Points to understand/derive:

- Snell's Law: \[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

- Internal Reflection

  This is known as the Goos-Hänchen effect and is similar to tunnelling.

  when \( \frac{n_1}{n_2} \sin \theta_1 > 1 \)

- Evaluate the forces on the interface, by evaluating the stress tensor

- How much light is reflected depends on the polarization of the incoming light.

  Depending on whether the magnetic or electric field points out of the scattering plane, (Transverse Magnetic or Transverse Electric - see handout) more or less light is reflected.
Thus unpolarized light will be partially polarized upon reflection.

This is used by radio towers to select transmitted light.
In this case the magnetic field points out of the scattering plane.
The transverse electric case is the magnetic dual of the transverse magnetic case.

\[ E \rightarrow B \]

\[ B \rightarrow -E \]

The E-field points into page (I to scattering plane)
We will study the Transverse Magnetic Case:
- Basic idea: write the solution in region II and region I as plane waves (sum of), and use boundary conditions to relate the two regions.

Region II: polarization vector
\[ \mathbf{E} = E_z e^{i k \cdot r - i \omega t} \]
\[ \mathbf{H} = H_z e^{i k \cdot r - i \omega t} (-\hat{y}) \] (out of page, transverse magnetic case)

Region I
\[ \mathbf{E}(t, \mathbf{r}) = E^I e^{i k \cdot r - i \omega t} + E^R e^{i k \cdot r - i \omega t} \]
\[ \mathbf{H}(t, \mathbf{r}) = (H^I e^{i k \cdot r - i \omega t} + H^R e^{i k \cdot r - i \omega t}) (-\hat{y}) \] (see figure!)

Boundary Conditions
\[ n \cdot (\mathbf{B}_2 - \mathbf{B}_1) = 0 \]
\[ n \times (\mathbf{H}_2 - \mathbf{H}_1) = 0 \] (Parallel components of \( \mathbf{H} \) continuous)
\[ n \cdot (\mathbf{B}_2 - \mathbf{B}_1) = 0 \]
\[ n \times (\mathbf{E}_2 - \mathbf{E}_1) = 0 \] (Parallel components of \( \mathbf{E} \) continuous)
Solving the B.C:

- Has to hold at all times and for every point on the interface:

\[ i \hat{k}_I \cdot \vec{r} - i\omega t \bigg|_{z=0} = i \hat{k}_R \cdot \vec{r} - i\omega t \bigg|_{z=0} = i \hat{k}_T \cdot \vec{r} - i\omega t \bigg|_{z=0} \]

- Frequencies have to be the same, so:

\[ \frac{\hat{k}_I}{\frac{\omega}{c}} = \frac{\hat{k}_R}{\frac{\omega}{c}} = \frac{\hat{k}_T}{\frac{\omega}{c}} \]

Thus the wavelengths are related:

\[ k_T = \frac{n_2}{n_1} k_I \]

- At \( z=0 \):

\[ \frac{\hat{k}_I}{\vec{r}} = k_x x = k \sin \theta x \]

so must have:

\[ k_I \sin \theta_1 = k_R \sin \theta_3 = k_T \sin \theta_2 \]

Or

\[ \Theta_1 = \Theta_3 \quad \text{incident = reflected} \]

\[ \sin \theta_1 = \frac{k_T \sin \theta_2}{k_I} \]

\[ \sin \theta_1 = \frac{n_2 \sin \theta_2}{n_1} \quad \text{(Snell's law)} \]
Now $E_\parallel$ continuous and $H_\parallel$ continuous

So the x-components of $E$ are continuous

$$-E_t \cos \theta_2 - (-E_t \cos \theta_1 + E_r \cos \theta_1) = 0$$

And from continuity of $H$

$$H_t - (H_t + H_r) = 0$$

So $H$ is related to $E$, $H = E/\omega$

$$+\frac{E_t}{\omega_2}, \quad -\frac{(E_t + E_r)}{\omega_1} = 0$$

Solving for the reflected and transmitted amplitudes:
Find:

\[
\frac{E_R}{E_I} = \frac{Z_1 \cos \theta_1 - Z_2 \cos \theta_2}{Z_1 \cos \theta_1 + Z_2 \cos \theta_2} \rightarrow \frac{Z_1 - Z_2}{Z_1 + Z_2}
\]

\[
\frac{E_I}{E_I} = \frac{2Z_2 \cos \theta_2}{Z_1 \cos \theta_1 + Z_2 \cos \theta_2} \rightarrow \frac{2Z_2}{Z_1 + Z_2}
\]

Now we want to analyze this:

- **Energy Transport**

\[
\vec{S} = \frac{c E \times H^*}{c} = \frac{\frac{1}{2} \frac{1}{Z} |E|^2 \vec{E}}{2}
\]

\[
\text{time averaged poynting flux}
\]

So the transmitted power, relative to the input power:

\[
\frac{P_T}{P_I} = \frac{\vec{S} \cdot \vec{n}}{\vec{S}_I \cdot \vec{n}} = \frac{\cos \theta_2 Z_2 |E_I|^2}{\cos \theta_1 Z_1 |E_I|^2}
\]

\[
\Rightarrow \frac{4Z_1 Z_2}{(Z_1 + Z_2)^2}
\]

\[
\frac{R_p}{P_I} = \frac{\vec{S}_R \cdot (-\vec{n})}{\vec{S}_I \cdot \vec{n}} = \frac{\cos \theta_1 Z_1 |E_R|^2}{\cos \theta_1 Z_1 |E_I|^2} \rightarrow \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}
\]
Final generally

\[ R + T = 1 \]

\[ \frac{P}{P} \]

**Momentum Transport:**

\[ n_2 = \text{something}, \quad \mu_2 = 1, \quad Z_2 = \frac{1}{n_2} \]

\[ n_1 = 1, \quad \mu_1 = 1, \quad Z_1 = 1 \]

\[ \text{Force} = - (T^{zz}_{out} - T^{zz}_{in}) \text{ over Area} \]

\[ H = \frac{E}{2} \]

\[ T^{zz} = \frac{E}{2} \left( - E_{t}^{2} + \frac{1}{E_{t}} \cdot E_{r}^{2} \cdot S^{zz} \right) + \frac{\mu}{2} \left( - H_{s}^{2} + H_{s}^{*} \cdot S^{zz} H_{s}^{*} \right) \]

\[ = \frac{E}{2} \left( E_{t}^{2} + \frac{1}{E_{t}} \cdot E_{r}^{2} \right) = \frac{E}{4} + \frac{1}{4} \frac{\mu}{E} \]

\[ S_0 \]

\[ T^{zz}_{out} = \frac{E_{t}^{2}}{2} E_{t}^2 = \frac{E_{t}^2}{2} \left( \frac{E_2}{E_1} \right) \left( \frac{E_T}{E_1} \right)^2 \]

\[ \frac{\text{index of refraction}}{\text{transmission}} \]

\[ \text{incident energy density} = \text{Coefficient} \]
Similarly

\[ T_{in}^{22} = \frac{3}{2} (E_I + E_R) \cdot (E_I + E_P) \]

\[ T_{in}^{22} = \frac{3}{2} \frac{E_I}{E_I} \cdot (1 + R) \]

\[ \langle u_I \rangle \]

Find

\[ \langle \text{Force} \rangle = \frac{\langle u_I \rangle}{\text{Area}} \left[ 1 + R - nT_p \right] \]

So for \( m \approx 1 \) and \( n = \frac{1}{z} \)

\[ \langle \text{Force/Area} \rangle = \frac{\langle u_I \rangle}{2(n-1)} \xrightarrow{n \to \infty} 2 \langle u_I \rangle \]

Note:

\[ \langle u_I \rangle = c \langle g_{em} \rangle = \langle S \rangle / c \]

\[ \begin{array}{c}
\text{energy density} \\sim \frac{m}{V} \\text{momentum} \\frac{\text{Energy}}{\text{Area} \cdot \text{time}} \\text{m/s}
\end{array} \]