Problem 1. Electric field in a dielectric shell.

A spherical dielectric shell, with a hollow interior, has inner radius $a$ and outer radius $b$ and dielectric constant $\varepsilon$. The dielectric shell sits in an external electric field of magnitude $E_o$ pointing in the $z$ direction.

(a) Find the system equations which determines the electric field inside the sphere, but (for lack of time) do not try to solve this system.

(b) Taking $a \to 0$ (so that the sphere is solid) determine the electric field within the sphere for $r < b$.

(c) It was not part of the exam, but solve for the electric field everywhere without using mathematica. Use two-by-two matrices to relate the solution in one region to another.

Problem 2. Induced rotation.

An infinitely long dielectric wire runs along the $z$ axis with charge density $\lambda$, and is surrounded by a thin dielectric cylindrical shell with radius $a$, carrying charge density $\sigma = -\lambda/(2\pi a)$. The suspended cylinder can rotate freely about the $z$-axis, but is initially at rest in a constant magnetic field in the $z$-direction, $B_{\text{ext}} = B_o \hat{z}$
(a) Determine the electric field for $t < 0$.

(b) At $t = 0$ we slowly reduce the magnetic field to zero over a time $T \gg a/c$. What happens and why? Draw a sketch of the resulting motion indicating the way that the cylinder rotates.

(c) Find the angular velocity of the cylinder as a function of time, taking its moment of inertia per unit length to be $I$.

(d) How did the condition $T \gg a/c$ help you in part (c) to find an approximate solution to the Maxwell equations. Point to a specific term in the Maxwell equations which was neglected/dropped/approximated using this condition. Give an estimate for the magnitude of the corrections to your result.

(e) Calculate the angular momentum per unit length for $t > T$ and show that it is conserved, i.e. that the final angular momentum equals the angular momentum for $t < 0$.

**Problem 3. Currents in a cylindrical shell.**

An infinite cylindrical shell of radius, $a$, carries a surface current in the $z$ direction which is a function of angle, $K(\phi) = K_o \cos(2\phi)\hat{z}$.

(a) Determine the magnetic field outside and inside the shell produced by the surface current.

(b) A second *small* circular loop lies carries current $I_o$ and has radius $r_o$ and sits on the $x$-axis at distance $\rho_o$ from the center. The current is oriented as shown. Determine the magnitude of the force on the current loop as a function of distance $\rho$ from the center of the cylinder. What is the direction of the force.
Problem 4. Reflection and transmission from glass

Consider a plane wave of light in vacuum normally incident (i.e. head on) on a semi-infinite slab of glass filling the space \( z > 0 \). The glass has index of refraction \( n > 1 \) and magnetic permeability \( \mu \simeq 1 \). The frequency of the light is \( \omega \). The incoming wave, is polarized in the \(-\hat{x}\) direction and has amplitude \( E_I \), thus taking the form

\[
E_I e^{-i\omega t + ikz} \hat{x}
\]  

(a) Starting from the Maxwell equations, determine the electric field amplitudes of the reflected and transmitted waves, in terms of the incident amplitude.

(b) Determine the reflection and transmission coefficients, i.e. the ratio of the reflected and transmitted power to the incident power.

(c) Determine the time averaged electromagnetic stress tensor both just in front, and just behind the vacuum-glass interface. Use this result to determine the force per unit area on the front face of the glass.