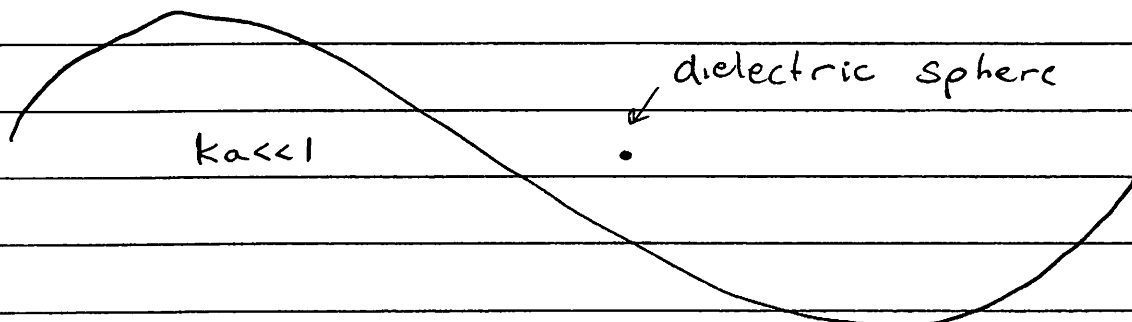
Dipole Scattering – Scattering By Small Objects

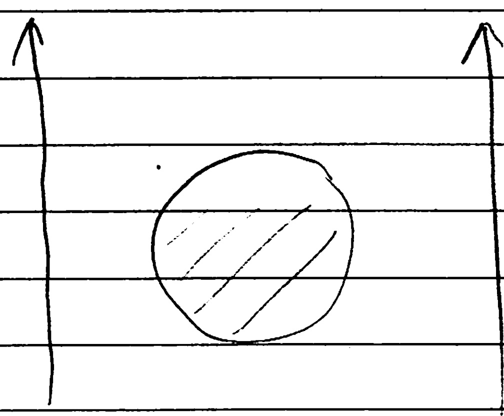
- Or why the sky is blue
- What means small? $ka \ll 1$

'Wave view



The small sphere experiences a uniform electric & magnetic field

Sphere view

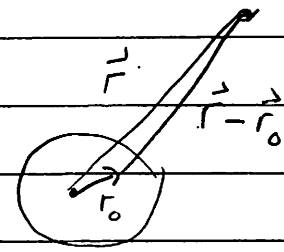


Electric field is constant and slowly varying

$$E = E_0 \vec{\mathcal{E}} e^{-i\omega t}$$

Dipole Scattering

The electric field induces a dipole moment which radiates. Lets quickly rederive the radiation field



$$\vec{A}_{rad} = \frac{1}{4\pi r}\int \frac{\vec{J}}{c}(T, r_0)\, d^3 r_0$$

$$T = t - \frac{r}{c} + \frac{n\cdot r_0}{c} \simeq t - \frac{r}{c}$$

So

$$A_{rad}(t, r) \simeq \frac{1}{4\pi r}\int \frac{\vec{J}}{c}\left(t - \frac{r}{c}, r_0\right) d^3 r_0$$

For a dipole at origin:

$$\vec{J} = \partial_t \vec{p}\;\; \delta^3(\vec{r}_0)$$

So

$$A_{rad}(t, r) = \frac{1}{4\pi r}\frac{1}{c}\,\dot{p}(t_e)$$

~~can also derive this~~ more formally see past lectures

Then we find

$$E_{rad} = n\times n\times \frac{\partial A_{rad}}{c\,\partial t}$$

$$= \frac{1}{4\pi r c^2}\, n\times n\times \ddot{p} = \frac{1}{4\pi r c^2}\left(-\ddot{p} + \vec{n}\,(n\cdot\ddot{p})\right)$$

Dipole Scattering pg. 2

Then the time averaged power radiated is

$$\frac{d\bar{P}}{d\Omega} = c\,\overline{(rE_{rad})^2}$$

$$= \frac{1}{16\pi^2 c^3}\overline{\left(-\ddot{\vec{p}} + \vec{n}\cdot(n\cdot\ddot{\vec{p}})\right)^2} = \frac{1}{16\pi^2 c^3}\overline{\left(\ddot{p}^2 - (n\cdot\ddot{p})^2\right)}$$

For a sinusoidal dipole moment $\vec{p} = p_\omega e^{-i\omega t}$ find

$$\frac{d\bar{P}}{d\Omega} = \frac{1}{16\pi^2 c^3}\frac{\omega^4}{2}\left(p_\omega\cdot p_\omega^* - (n\cdot p_\omega)(n\cdot p_\omega^*)\right)$$

from time ave

The induced dipole moment is proportional to incoming field

$$\vec{p} = \alpha_E \vec{E}_{inc} \qquad \alpha_E = 4\pi\left(\frac{\varepsilon-1}{\varepsilon+2}\right)a^3$$

polarizability

found by solving for the induced charges on a dielectric sphere in a const field,

So with $\vec{E}_{inc} = \vec{\varepsilon}_0 E_0 e^{-i\omega t}$

$$\vec{p} = \underbrace{\alpha_E E_0 \varepsilon_0}_{\equiv p_\omega} e^{-i\omega t}$$

And

$$\frac{d\bar{P}}{d\Omega} = \frac{1}{16\pi^2 c^3} \omega^4 \frac{1}{2}\alpha_E^2 E_0^2 \left(1 - (n\cdot\varepsilon_0)(n\cdot\varepsilon_0^*)\right)$$

The cross section is the averaged power by the incoming flux

$$\frac{d\sigma}{d\Omega} = \frac{d\bar{P}/d\Omega}{\frac{1}{2}cE_0^2}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2}\left(\frac{\omega}{c}\right)^4 \alpha_E^2 \left(1 - |n\cdot\varepsilon_0|^2\right)}$$

Or

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{\varepsilon-1}{\varepsilon+2}\right)^2 \left(\frac{\omega a}{c}\right)^4 a^2 \left(1 - |n\cdot\varepsilon_0|^2\right)}$$

Important Remarks

- See a characteristic frequency dependence to dipole Scattering

$$\sigma \propto \omega^4$$

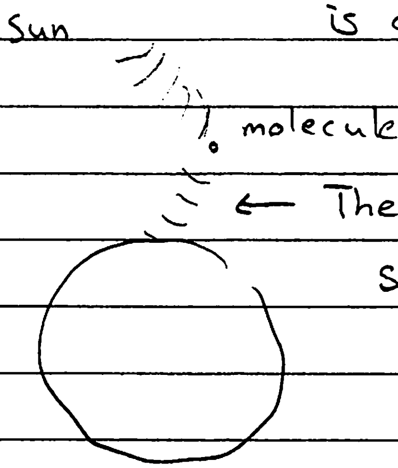
- Dimensions fix the remaing factors

$$\sigma \propto \left(\frac{\omega a}{c}\right)^4 a^2$$

Why Sky is Blue?

$\sigma \propto \omega^4$ so most of the scatterred light is at high frequency.

Midday



← The higher frequencies are preferentially scattered towards our eyes.
Higher frequencies = bluer

At **Sunset**, the blue light is scattered away and only the red light traverses the atmosphere to reach our eyes