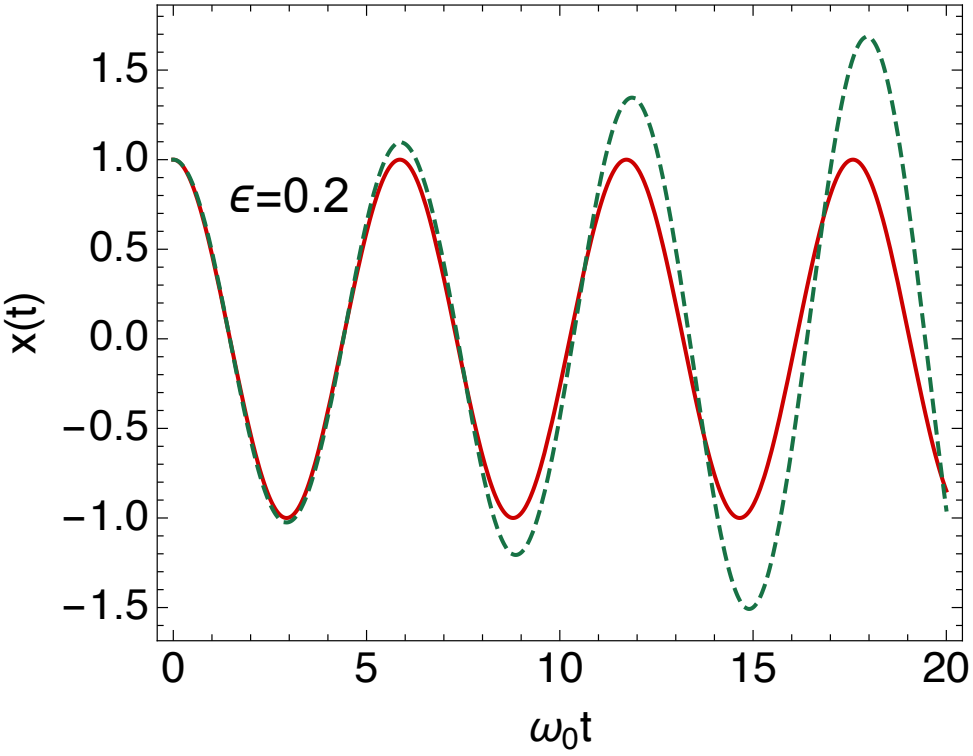
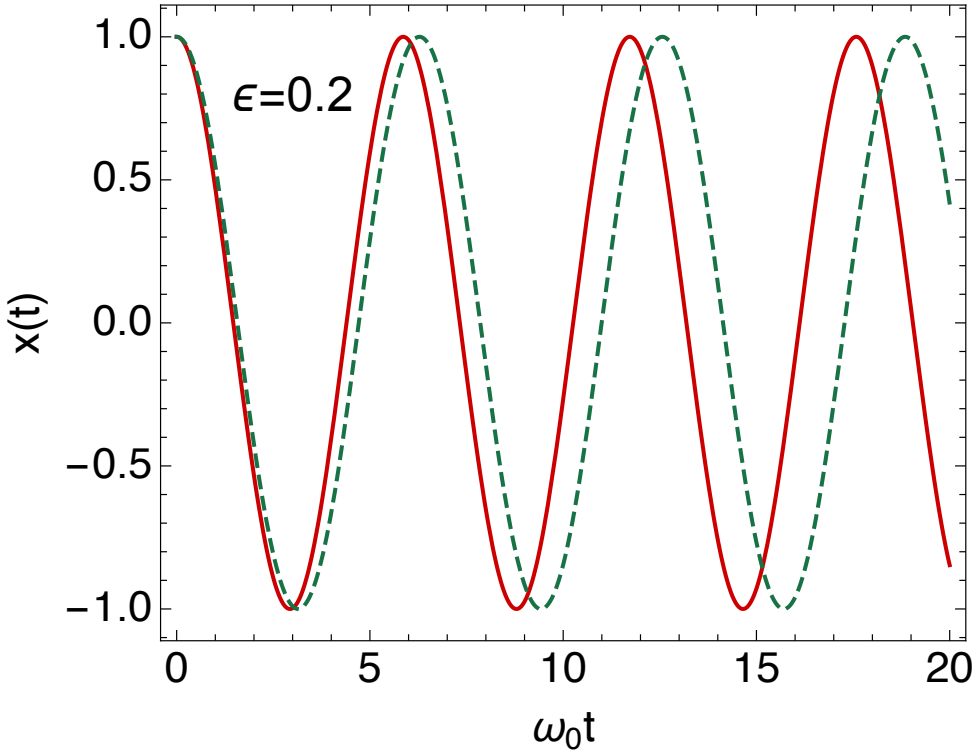
Nonlinear Oscillations

Naive Perturbation Theory: Diverges after t=1/epsilon

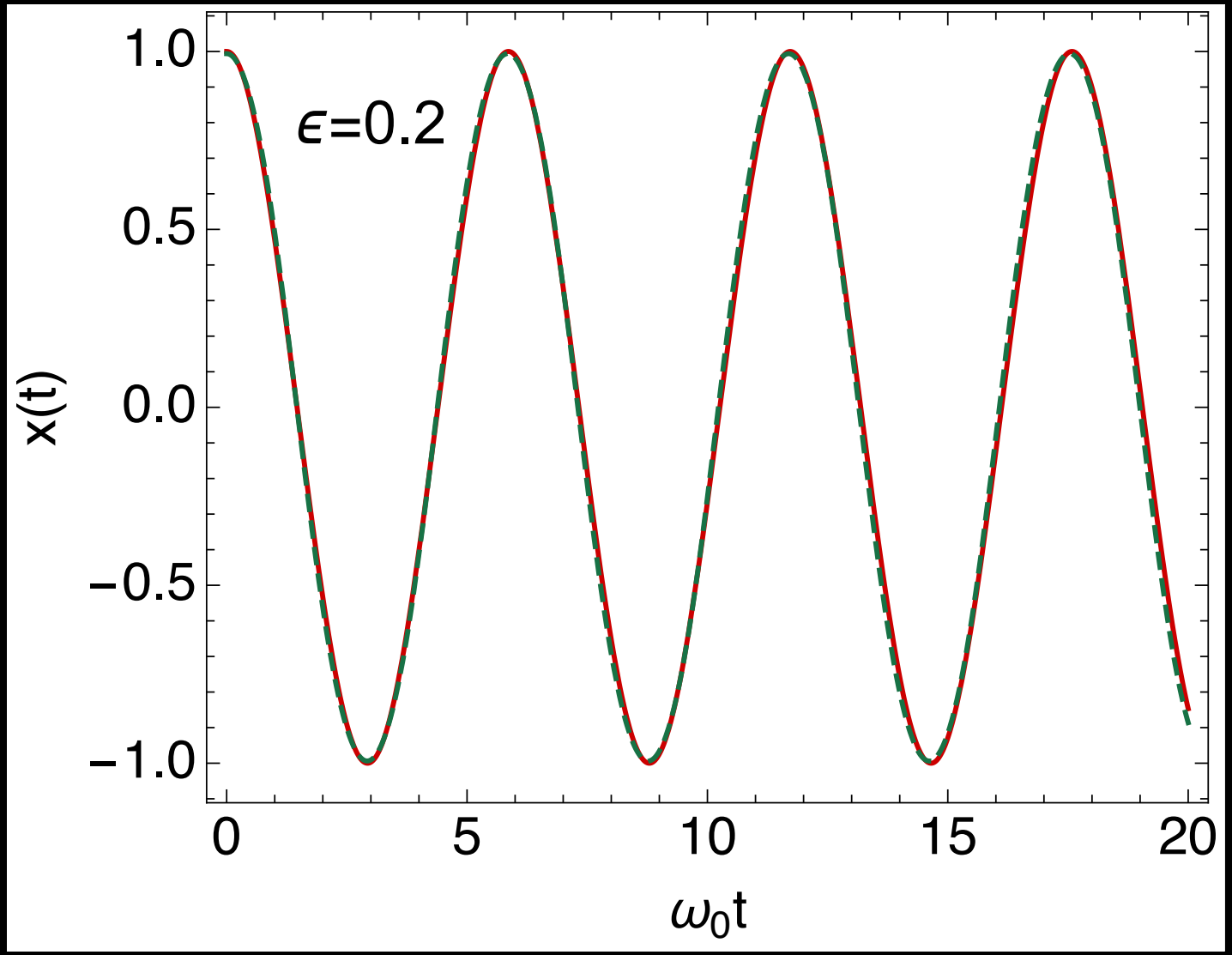
Non linear oscillator
with secular terms



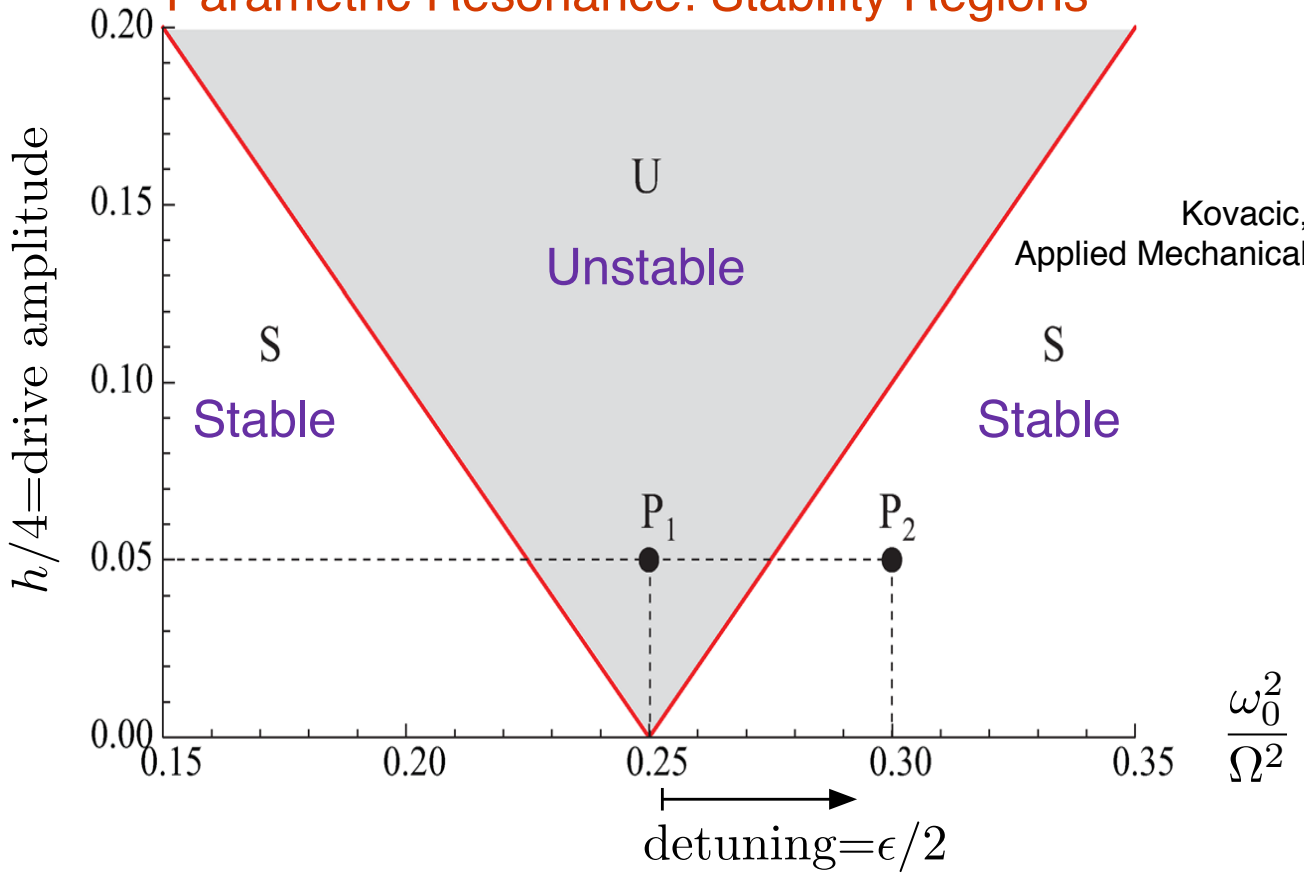
Non linear oscillator
w/out secular terms



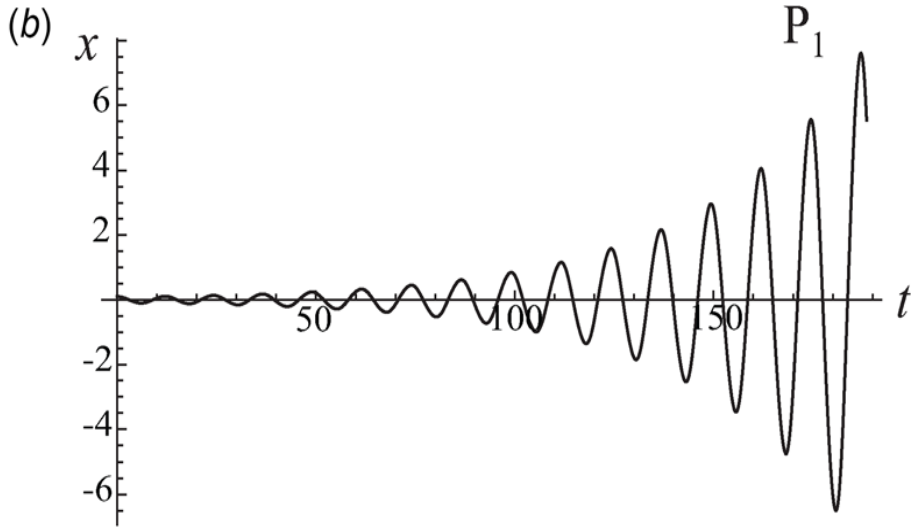
Secular Perturbation Theory works great! Here $\Delta\omega = 3\beta a^2 / 8\omega_0$



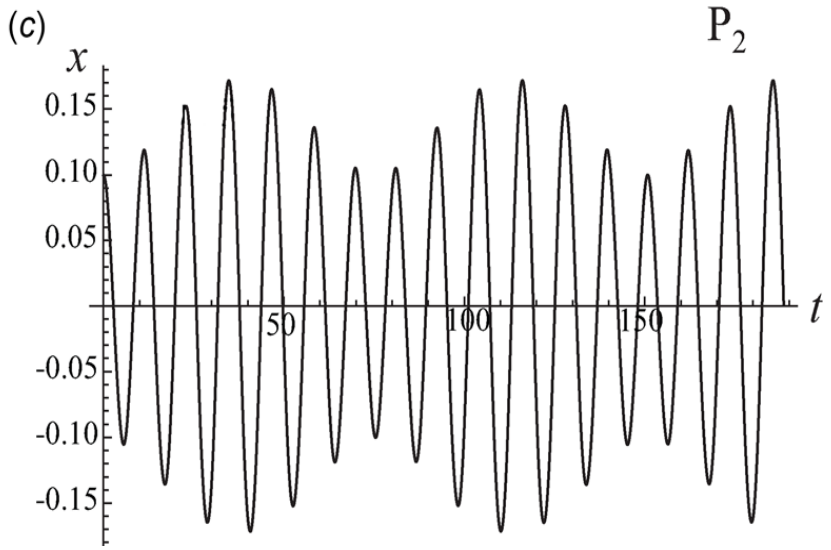
Parametric Resonance: Stability Regions



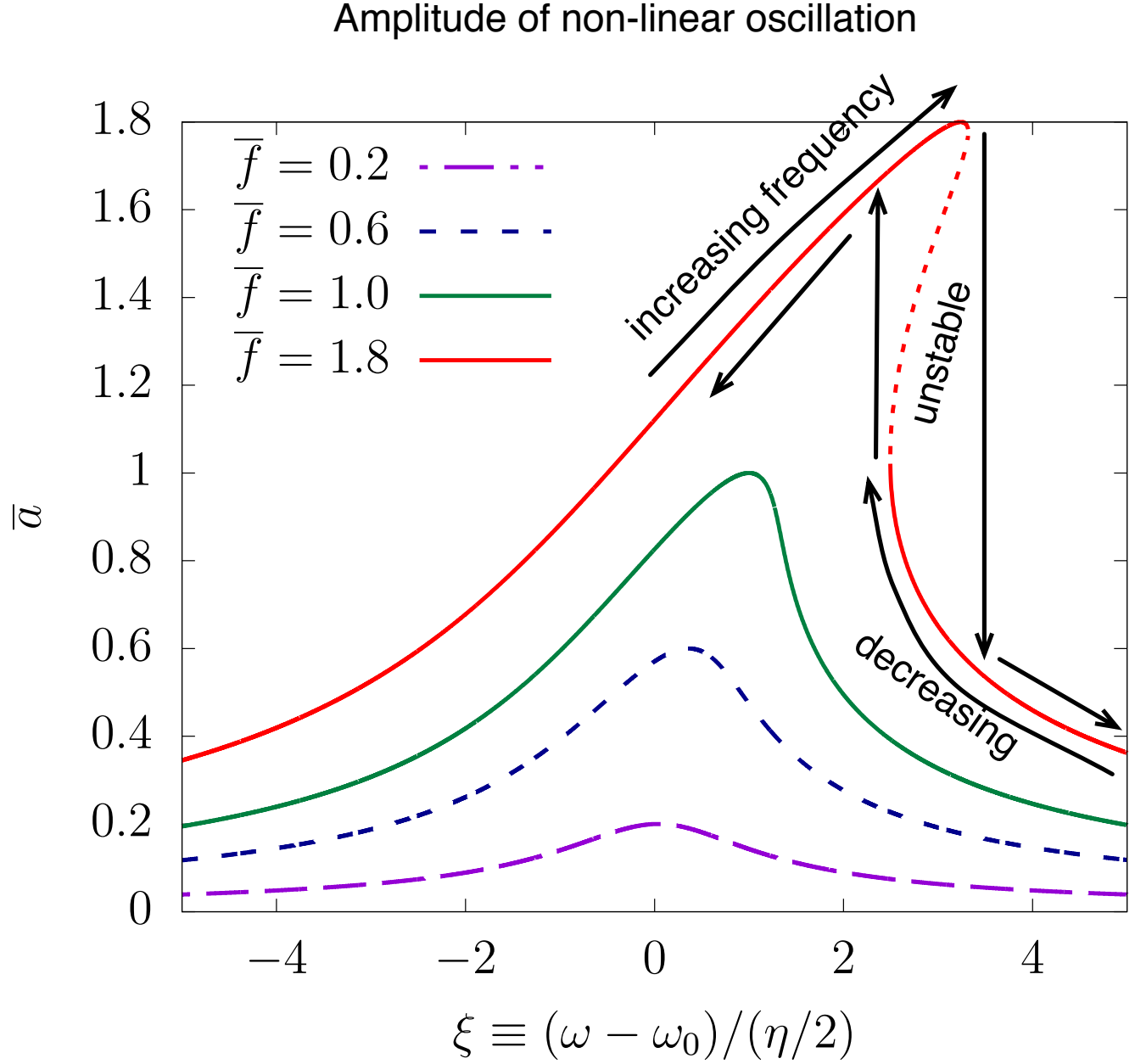
Kovacic, Rand, Sah
Applied Mechanical Review, 70, 020802-1



Unstable oscillations at P1



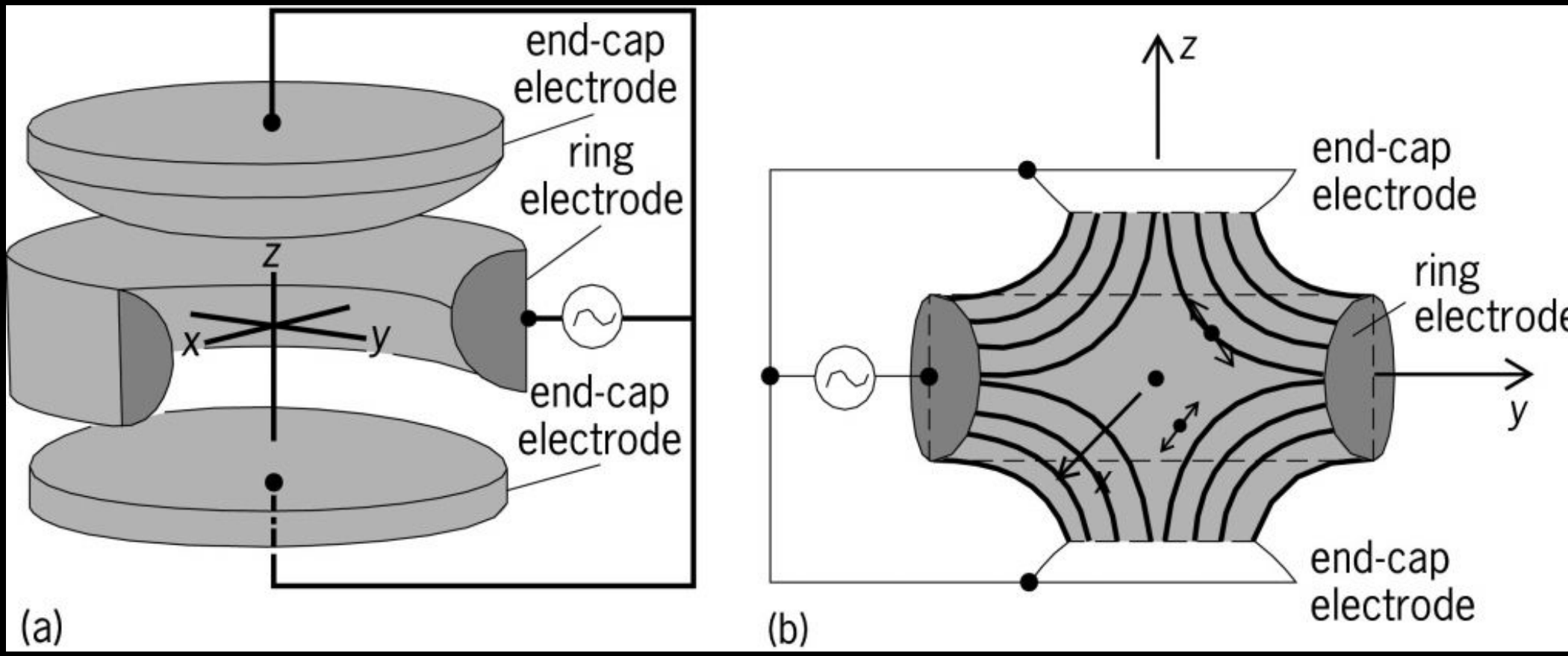
Stable oscillations at P2


Amplitude of non-linear oscillation
$\overline{f} = 0.2$
$\overline{f} = 0.6$
$\overline{f} = 1.0$
$\overline{f} = 1.8$
increasing frequency
unstable
decreasing
$\overline{a}$
$\xi \equiv (\omega - \omega_0)/(\eta/2)$
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−4
−2
0
2
4

Inverted Pendulum:

Click me

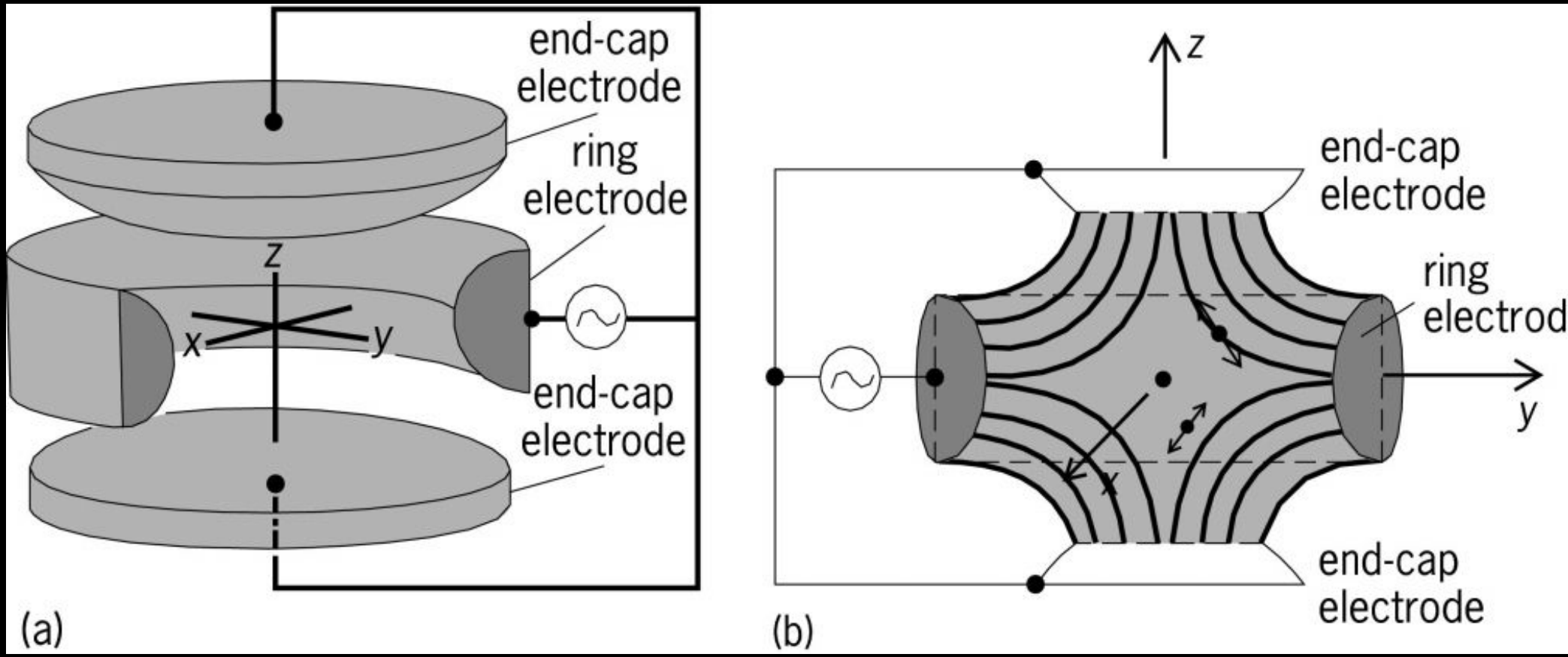
A Paul Trap for Ion Trapping



Electrostatic Potential

$$\phi = \frac{1}{2} A(x^2 + y^2 - 2z^2)$$

A Paul Trap for Ion Trapping



Electrostatic Potential

$$\phi = \frac{1}{2} A \cos(\Omega t)(x^2 + y^2 - 2z^2)$$

The ponderamotve potential is:

$$U_{\text{eff}} = \frac{q^2 A^2}{2m\Omega^2}(x^2 + y^2 + 4z^2)$$