Gallilean Transformations

- Consider an isolated system $U^{ext} = 0$ and for simplicity consider just two particles

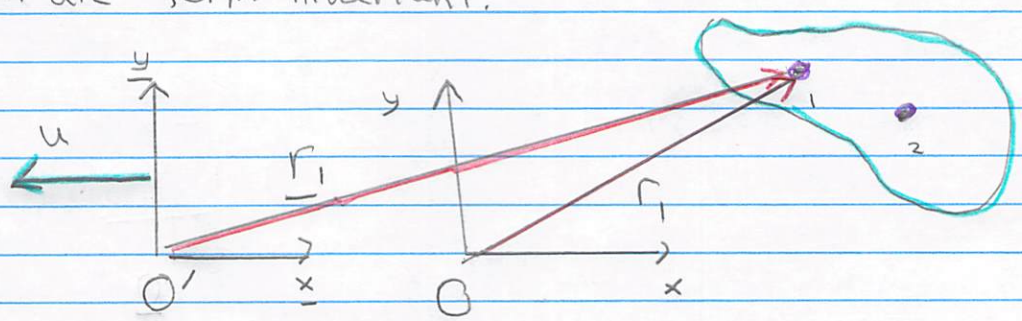
(1) $$\frac{d\vec{p}_a}{dt} = -\frac{\partial U}{\partial \vec{r}_a} \qquad \vec{p}_a = m\vec{v}_a$$

Where

$$U = U^{int}(|\vec{r}_1 - \vec{r}_2|)$$

- The system is measured by two observers who set up there own space-time coordinate systems (Frames), which are O and O'.

O' is moving with constant velocity $-\vec{u}$ relative to O. They must have the same EOM! The EOM are form invariant.



- Their coordinates are related

$$\vec{r}_a \longrightarrow \vec{r}' = \vec{r}_a + \vec{u}\,t \quad \longleftarrow \text{Gallilean Boost}$$

$$t \longrightarrow t' = t$$

- The potential is unchanged by the shift

$$U(|\vec{r}_1 - \vec{r}_2|) = U(|\vec{r}_1' - \vec{r}_2'|)$$

- But the velocities and momenta are shifted by a <u>constant</u>

$$\vec{V}_a \longrightarrow \vec{V}_a' = \vec{V}_a + \vec{u}$$

$$\vec{p}_a \longrightarrow \vec{p}_a' = \vec{p}_a + m_a \vec{u}$$

Thus, the equation's of motion are unchanged by the Gallilean boost.

$$\boxed{\frac{d\vec{p}_a'}{dt'} = -\frac{\partial U}{\partial \vec{r}_a'}}$$

this is the same form as Eq. (1), and thus the EOM are <u>frame</u> invariant