Fermi Gasses Again

1D $\quad E_k = \frac{\hbar^2 k^2}{2m} \qquad k_n = \frac{n\pi}{L}$ } particle in box

$\Delta k = \frac{\pi}{L}$

Fill states up to k_F

The total number of particles is

$$N = \int_0^{k_F} \frac{dk}{\Delta k} = \int_0^{k_F} \frac{dk}{\frac{\pi}{L}} = \frac{L}{\pi} k_F$$

$$\boxed{\left(\frac{N}{L}\right) = \frac{k_F}{\pi}}$$

number of partices / Length

$$E = \sum_n \frac{\hbar^2 k_n^2}{2m} = \int_0^{k_F} \frac{dk}{\Delta k} \frac{\hbar^2 k^2}{2m}$$

$$E = \int_0^{k_F} L \frac{dk}{\pi} \frac{\hbar^2 k^2}{2m} = L \frac{\hbar^2}{6m\pi} k_F^3$$

$$\frac{E}{N} = \text{Energy per particle}$$

$$\frac{E}{N} = \frac{\frac{L}{\pi} \frac{\hbar^2}{2m} \frac{1}{3} k_F^3}{\frac{L\, k_F}{\pi}}$$

$$\boxed{\frac{E}{N} = \frac{1}{3} \cdot \left(\frac{\hbar^2 k_F^2}{2m} \right)}$$

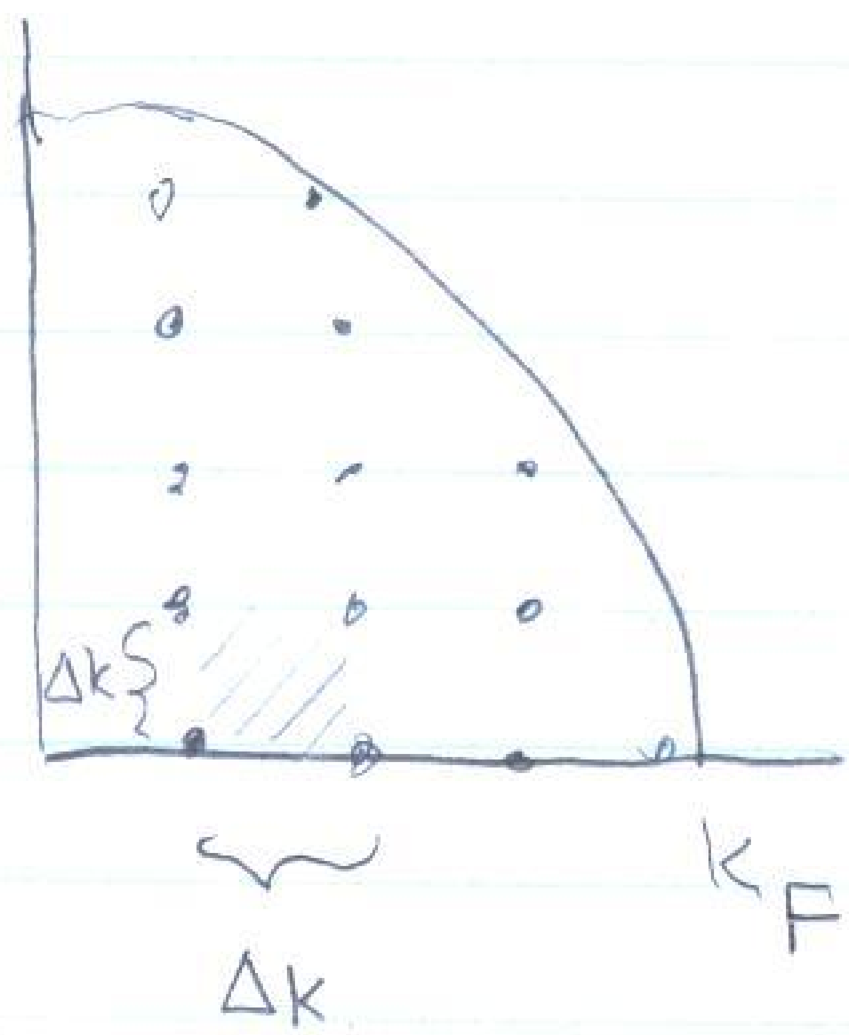
Fermi Energy
= Energy at top

2D

Particle in Box

$$E = \frac{\hbar^2}{2m}(k_x^2 + k_y^2) \qquad k_x = n_x \frac{\pi}{L}$$

$$k_y = n_y \frac{\pi}{L}$$



$$(\Delta k)^2 = \text{size of cell}$$

$$= \left(\frac{\pi}{L}\right)^2$$

The number of states within the fermi sphere is

number of particles with spin up, say.

$$N = \int_{\frac{1}{4}\text{ Sphere}} \frac{dk_x\, dk_y}{(\Delta k)^2} = \int_{\frac{1}{4}\text{ spere}} \frac{L^2 d^2k}{\pi^2} = \int_0^{k_F} \frac{L^2}{\pi^2}\left(\frac{2\pi k}{4}\right) dk$$

integration over $\frac{1}{4}$ of sphere

$$N = \frac{L^2}{\pi^2} \cdot \frac{\pi}{2} \cdot \frac{k_F^2}{2}$$

$$\frac{N}{A} = \frac{\text{number of particels}}{\text{Area}}$$

$$\boxed{\frac{N}{A} = \frac{k_F^2}{4\pi}}$$

$$\boxed{\left(4\pi \frac{N}{A}\right)^{1/2} = k_F}$$

$$E = \sum_n \frac{\hbar^2 \vec{k}^2}{2m}$$

$$E = \int_{\frac{1}{4}\text{ Sphere}} \frac{dk_x\, dk_y}{(\Delta k)^2} \frac{\hbar^2 k^2}{2m}$$

$$E = \int_{\frac{1}{4}\text{ sphere}} \frac{L^2}{\pi^2} d^2k \frac{\hbar^2 k^2}{2m}$$

$$E = \int_0^{k_F} \frac{L^2}{\pi^2} \left(\frac{2\pi k}{4}\right) dk \frac{\hbar^2 k^2}{2m}$$

$$E = \frac{L^2}{2\pi} \frac{\hbar^2}{2m} \int_0^{k_F} k^3$$

$$E = L^2/2\pi \; \frac{\hbar^2}{2m} \; \frac{k_F^4}{4}$$

$$\frac{E}{N} = \frac{2}{4}\frac{\hbar^2 k_F^2}{2m} = \frac{1}{2}\,\varepsilon_F$$

Fermi Energy $\frac{\hbar^2 k_F^2}{2m}$

3D

You do it!

$$\boxed{\left(6\pi^2 \frac{N}{V}\right)^{1/3} = k_F}$$

$$\boxed{\frac{N}{V} = \frac{k_F^3}{6\pi^2}}$$

$$\boxed{\frac{E}{N} = \frac{3}{5}\frac{\hbar^2 k_F^2}{2m}}$$

N/V is the number of say spin up identical particles (say protons) per

$$\frac{N}{V} = \frac{1}{2}\cdot\frac{1}{2}\underbrace{\rho_0}_{\text{nuclear}} = \frac{1}{4}\cdot\frac{1}{6}\frac{1}{fm^3} = \frac{1}{24}\frac{1}{fm^3}$$

matter density = # of protons or neutron with spin ↑ or ↓ per volume

$= \frac{1}{6}\frac{1}{fm^3}$

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