Last Times :

For Large nuclei

- $\rho = \dfrac{A}{\frac{4}{3}\pi R^3} \longrightarrow \text{constant} = \dfrac{1}{6}\dfrac{\text{nuc}}{\text{fm}^3}$

- means that instead of Considering finite nuclei, can consider nuclear matter

We Looked At The masses

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Compare (neglect mass diff between protons & neutro

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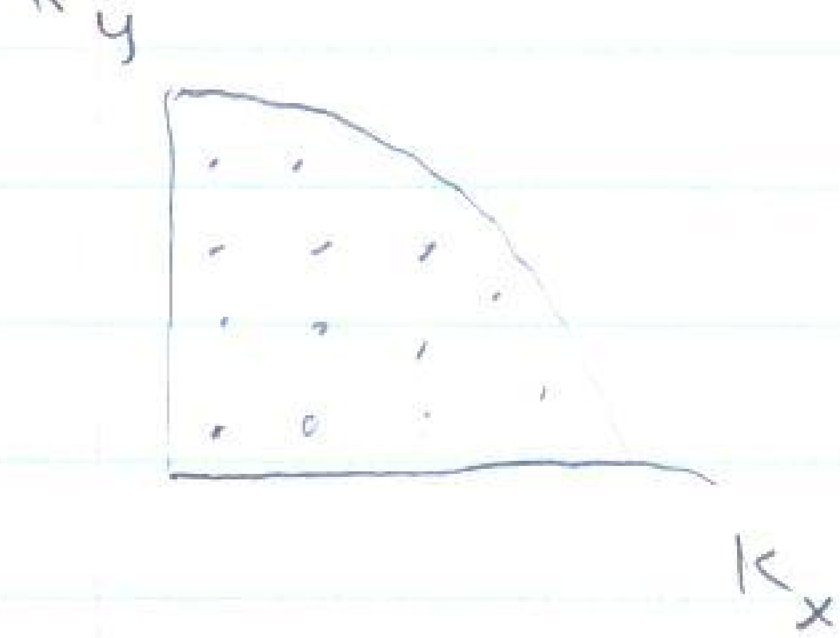
We ~~evaluated~~ estimated the Kinetic Energy per particle with the Fermi-gas



For fixed Volume and fixed Number we fill all states up to the fermi momentum

$$\left(\frac{N}{V}\right)^{1/3} \propto \frac{1}{\Delta x} = \frac{1}{\text{distance between particles}}$$

$$\frac{p_F}{\hbar} \approx \frac{1}{\Delta x} \propto \left(\frac{N}{V}\right)^{1/3}$$

<u>Exactly</u> $$\frac{p_f}{\hbar} = \left(6\pi^2 \frac{N}{V}\right)^{1/3}$$

$\leftarrow$ density of say protons with spin up

$$\frac{N}{V} = \frac{1}{4} \cdot \frac{1}{6} \, \frac{\text{Nucleons}}{fm^3}$$

$$p_F \approx 200 \text{ MeV}$$

$$E_F = \frac{p_f^2}{2m} = 35 \text{ MeV}$$

Finally used the Fermi Gass model

$$\left\langle \frac{KE}{A} \right\rangle \simeq \frac{3}{5} \varepsilon_F = 21 \text{ MeV}$$

Then

$$\underbrace{-B}_{-8\text{MeV}} = \underbrace{\frac{KE}{A}}_{21\text{MeV}} + \underbrace{\frac{PE}{A}}_{-29\text{MeV}}$$

Ultra Important:

- Nuclei are The result of a sharp cancellation of Kinetic and Potential Forces

$$\frac{B}{A} \simeq \underbrace{a_v}^{\sim 8{,}5\,MeV}$$

$$B = a_v A - \underbrace{a_s A^{2/3}}_{\text{Surface}} - \underbrace{a_c \frac{Z(Z-1)}{A^{1/3}}}_{\substack{\text{Coulomb} \\ 0.72\,MeV}}$$

$$+ a_{sym} \frac{(A-2Z)^2}{A} + \delta$$

tends to make number of neutrons = protons

Nuclei like to have even # of protons and neutrons
→ well get to it

$a_v = 15.8\ MeV$

$a_s = 17.8\ MeV$

$a_c = 0.71\ MeV$

$a_{sym} = 23\ MeV$

Pairing

- There are four nuclei with odd N and Z
  ^{2}H, ^{6}Li, ^{10}B, ^{14}N

- 167 species with even N and Z

$$\delta = \begin{cases} +a_p A^{-3/4} & Z \text{ even} - N \text{ even} \\ 0 & \text{even} - \text{odd} \\ -a_p A^{-3/4} & \text{odd} - \text{odd} \end{cases}$$

$$B = a_V A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{A} + \delta$$

Then

$$M(Z,A) = Z\, m(^{1}H) + N m_N - B(Z,A)/c^2$$

$$Z_{min} = \frac{[m_N - m(^{1}H)] + a_c A^{-1/3} + 4a_{sym}}{2a_c A^{-1/3} + 8a_{sym}/A}$$

$$= \left.\frac{\partial M}{\partial Z}\right|_{A \text{ fixed}}$$

$$Z_{min} \simeq \frac{A}{2} \frac{1}{\left(1 + \frac{1}{4} A^{2/3} a_c / a_{sym}\right)}$$

Then this explains the following

- Small nuclei have $Z \approx \frac{A}{2}$

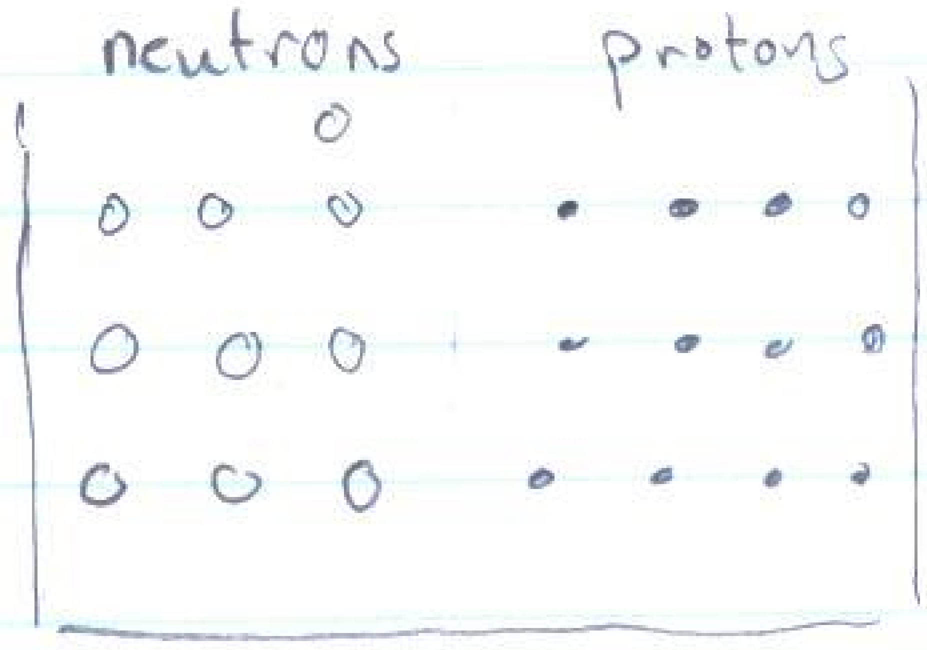
- As the nucleus grows number of neutrons is larger than number of protons to offset the increase of coulomb

- heavy nuclei, approximately correct

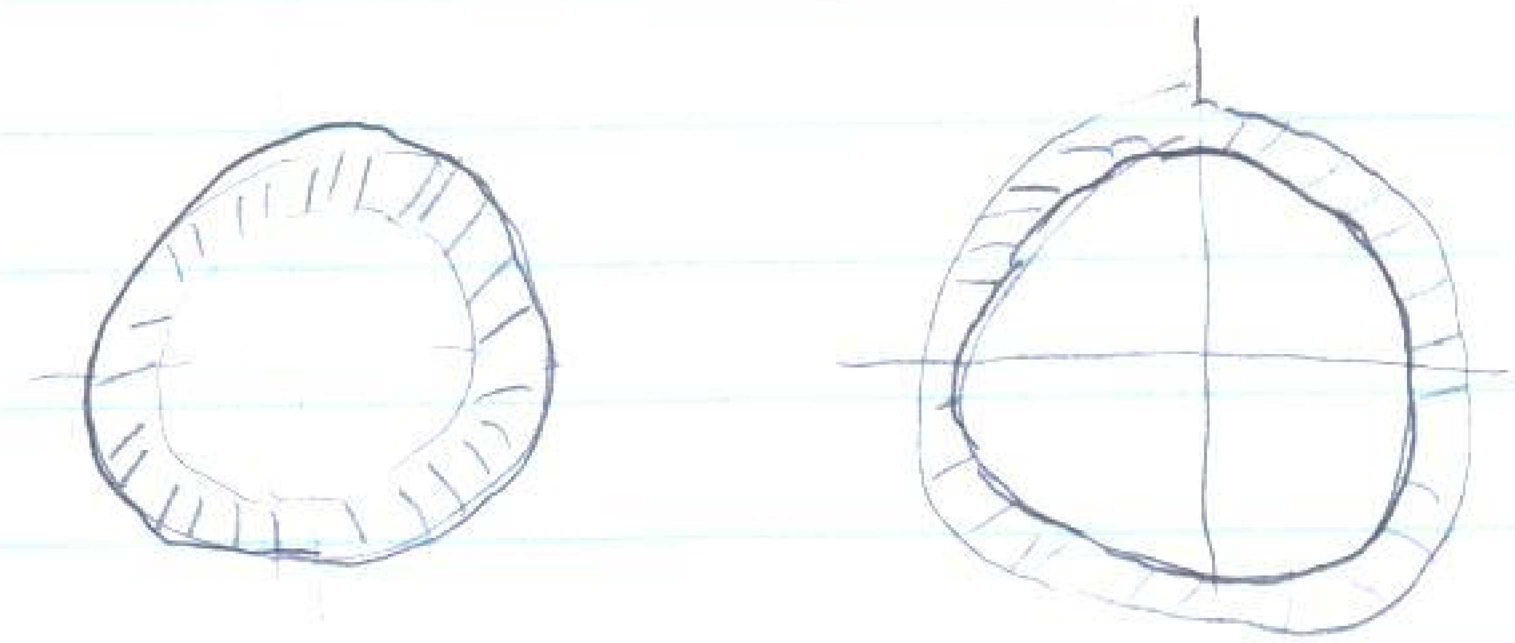
$$\frac{Z}{A} \approx 0.41$$

$$\frac{Z_{min}}{A/2} \simeq \frac{1}{\left(1 + \frac{1}{4} A^{2/3} a_c / a_{sym}\right)} = 0.41 \Bigg|_{A=140}$$

Then consider



If we convert a proton to a neutron then we will add to the fermi see



$$N = \left(\frac{g}{2\pi^2}\right)\frac{1}{3}\left(\frac{p_f}{\hbar}\right)^3 \cdot V \qquad E = \frac{g}{2\pi^2}\,\frac{1}{3}\left(\frac{p_f}{\hbar}\right)^3\left(\frac{3}{5}\,\frac{p_f^2}{2m}\right)\cdot V$$

$$\left(\frac{6\pi^2}{g}\,\frac{N}{V}\right)^{1/3} = \frac{p_f}{\hbar}$$

$$E = V\,C_E\,p_f^5 \qquad N = V\,C_N\,p_f^3$$

Then

$$\Delta E_{TOT} = \Delta E_p + \Delta E_N = 2\left[\overbrace{\frac{1}{2}\frac{\partial^2 E}{\partial p_f^2}(\Delta p_F)^2}^{\Delta E_p} + \overbrace{\frac{1}{2}\frac{\partial^2 E}{\partial p_f^2}(\Delta p_f)^2}^{\Delta E_N}\right]$$

$$= \frac{\partial^2 E}{\partial p_f^2}(\Delta p_F)^2$$

$$E = V \cdot C_E \, p_f^5 \qquad C_E = \frac{g}{2\pi^2}\frac{1}{5}\cdot\frac{1}{2m}\frac{1}{\hbar^3}$$

$$\frac{\partial^2 E}{\partial p_f^2} = 5\cdot 4\cdot V C_E \, p_f^3$$

$$\Delta N = \frac{\partial N}{\partial p_f}\Delta p_f = 3C_N \cdot V \, p_f^2 \, \Delta p_f$$

$$\Delta E_{TOT} = 20 \, C_E \cdot V \, p_f^3 \cdot \left[\frac{\Delta N}{(3C_N \cdot V \cdot p_f^2)}\right]^2$$

$$\Delta E_{TOT} = \frac{20}{9}\frac{C_E \cdot \cancel{V}}{C_N^2 \, V^{\cancel{2}}}\frac{1}{p_f}(\Delta N)^2 \qquad p_f =$$

$$\Delta E_{TOT} = \frac{20}{9}\frac{C_E}{C_N^2}\underbrace{\frac{1}{(p_f)}}_{\text{Const}}\frac{(\Delta N)^2}{V}$$

$$\Delta N = \left(\frac{N-Z}{2}\right)$$

$$V = \frac{A}{\rho_0}$$

So $$\Delta E_{TOT} = \frac{20}{9}\frac{C_E}{C_N^2}\frac{1}{p_f}\,\frac{1}{4}\cdot\frac{(N-Z)^2}{\frac{A}{\rho_0}}$$

$$C_N p_f^3 = \frac{\rho_0}{2} \longrightarrow$$

$$\Delta E_{TOT} = \frac{20}{9}\frac{C_E}{C_N^2}\cdot\frac{1}{p_f}\,\frac{1}{4}\overbrace{\rho_0}^{2C_N p_f^3}\frac{(N-Z)^2}{A}$$

$$\Delta E_{TOT} = \frac{10}{9}\frac{C_E}{C_N}p_f^2\,\frac{(N-Z)^2}{A}$$

$$\Delta E_{TOT} = \frac{10}{9}\underbrace{\frac{p_f^2}{2m}}_{\sim 25\,\text{MeV from before}}\frac{(N-Z)^2}{A}$$

$$\Delta E \sim 27\ \text{MeV}\left(\frac{N-Z}{A}\right)^2$$

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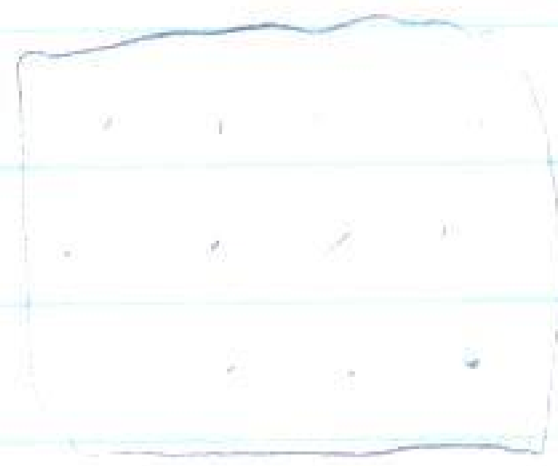

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