Last Time:

$$f(x) = \sum_{k} \phi_k(x)\, f(k) \qquad \phi_k = \frac{e^{+ik\cdot x}}{\sqrt{L}}$$

$$k = \frac{2\pi n}{L}, \quad n = -\infty, \ldots$$

$$f(k) = \int_{-L}^{L} \frac{e^{-ikx}}{\sqrt{L}} f(x)$$

This follows because $\int_L \phi_k^* \phi_{k'} = \delta_{kk'}$

k

$\phi = \frac{1}{\sqrt{L}}$

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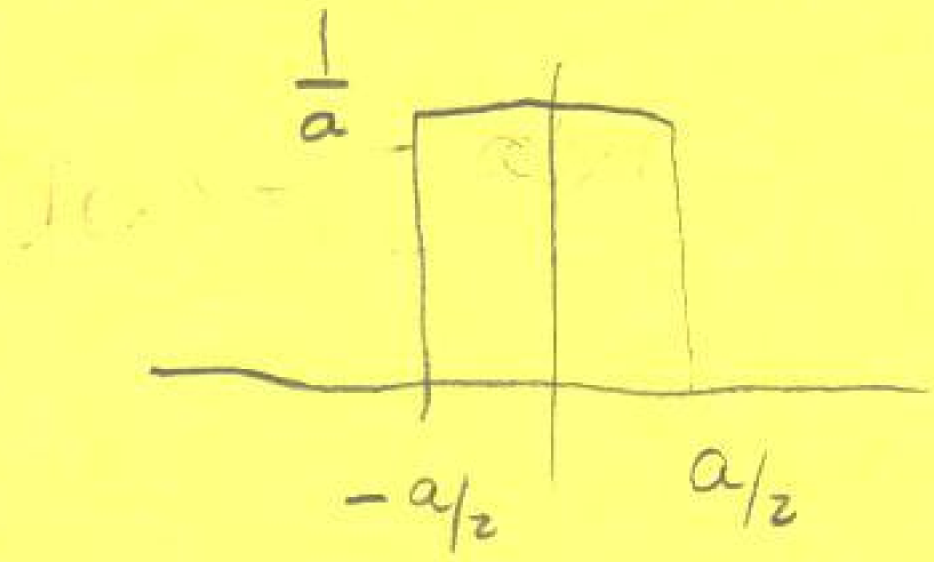
Then consider the Function

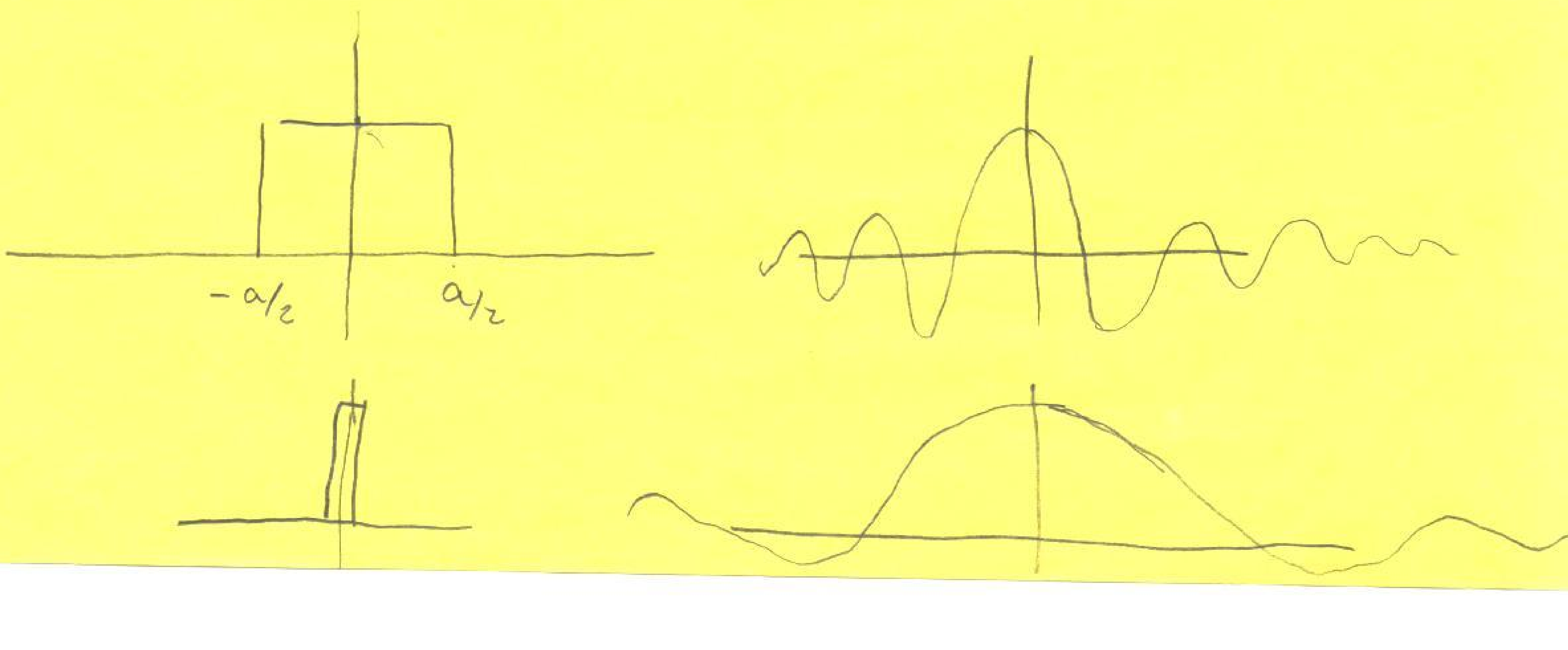


$$f(k) = \int_{-L}^{L} dx \; \frac{e^{-ikx}}{\sqrt{L}} \; \overbrace{f(x)}^{\frac{1}{a}} = \frac{1}{\sqrt{L}} \; \frac{e^{-ikx}}{-ika} \Bigg|_{-a/2}^{a/2}$$

$$f(k) = \frac{1}{\sqrt{L}} \; \frac{e^{-ika/2} - e^{+ika/2}}{-ik}$$

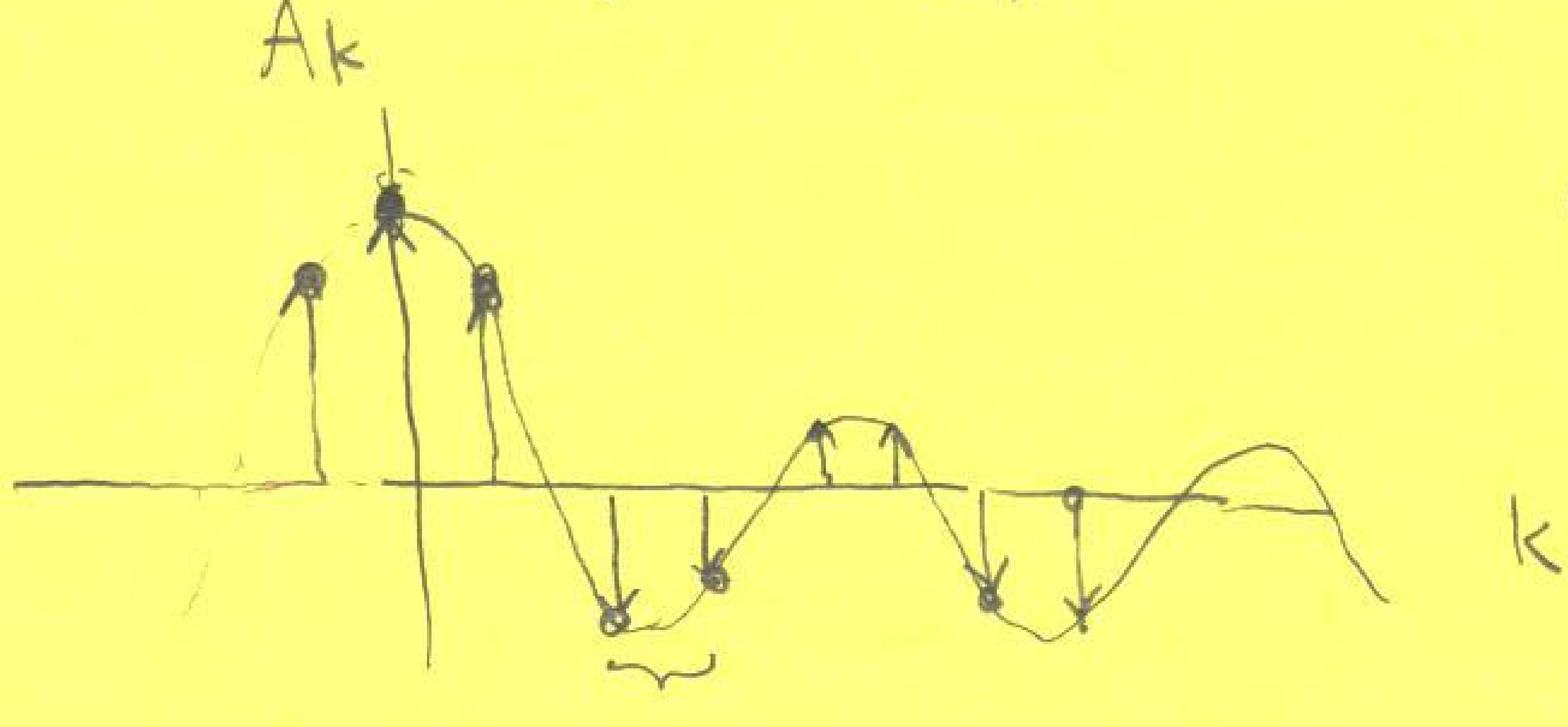
$$f(k) = \frac{1}{\sqrt{L}} \cdot 2 \; \frac{e^{ika/2} - e^{-ika/2}}{2i\,ka}$$

$$f(k) = \frac{1}{\sqrt{L}} \; \frac{\sin(ka/2)}{ka/2} \longrightarrow f_{kont}(k)$$



Take

$$f_k = \frac{1}{\sqrt{L}} \frac{\sin ka/2}{ka/2} \qquad k = \frac{2\pi n}{L}$$



C_k is defined at disc
points

$$\Delta k = \frac{2\pi}{L}$$

Take $L \to \infty$ the spacing between points $\to 0$

- $$f(x) = \sum_k \frac{e^{ik \cdot x}}{\sqrt{L}} f(k)$$

- $$\sum_n = \int dn = \int L \frac{dk}{2\pi} \qquad k = \frac{2\pi n}{L}$$

- $$f(x) = \int_{-\infty}^{\infty} \frac{L\,dk}{2\pi} \frac{e^{+ik \cdot x}}{\sqrt{L}} f_{box}(k)$$

- $$f_{box}(k) = \int_{-\infty}^{\infty} dx \frac{e^{-ik \cdot x}}{\sqrt{L}} f(x)$$

Need to distinguish

define $$\boxed{f_{cont}(k) = \int_{-\infty}^{\infty} dx\, e^{-ik\cdot x} f(x)}$$

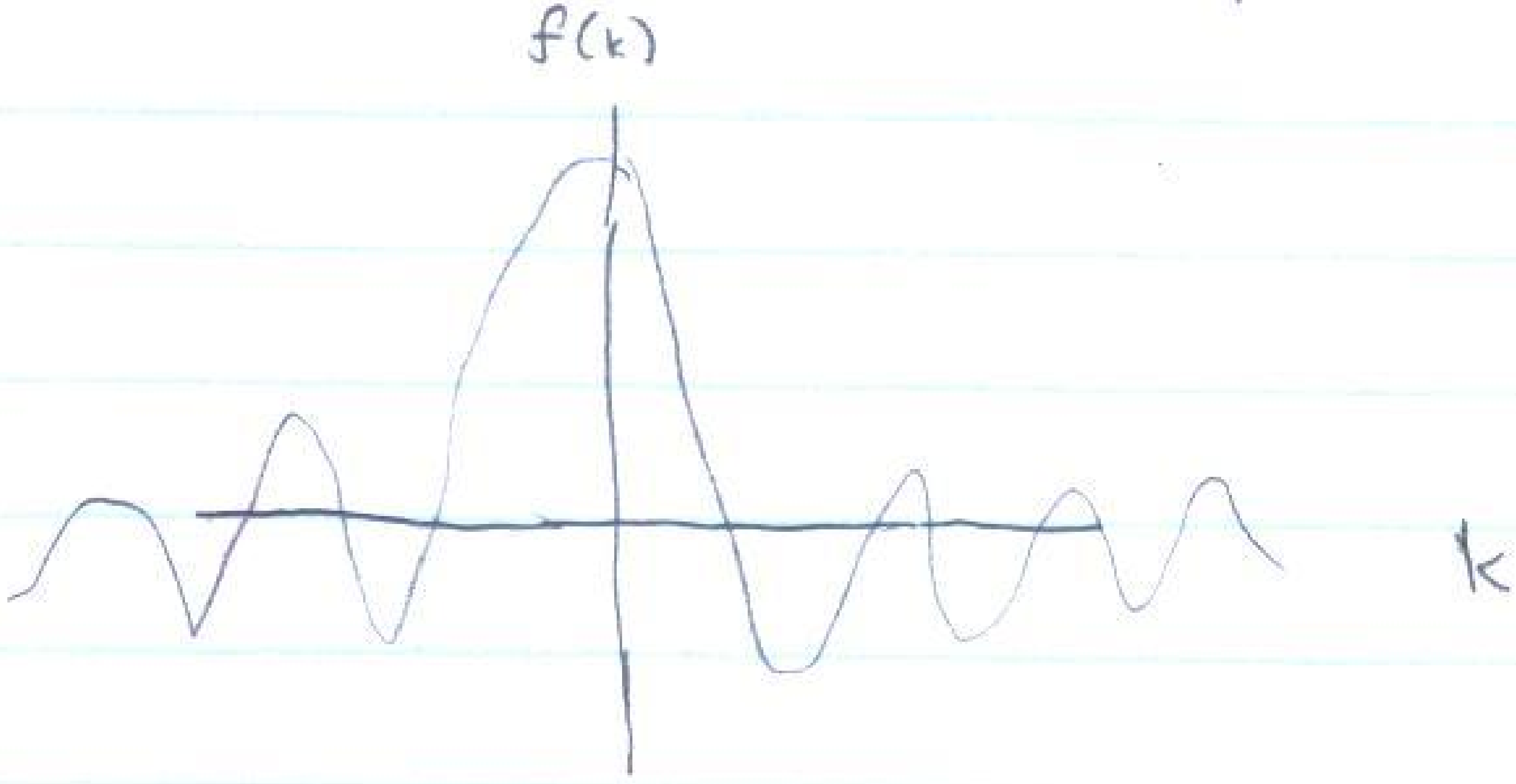
$$\boxed{f_{box}(k) = \frac{f_{cont}(k)}{\sqrt{L}}}$$

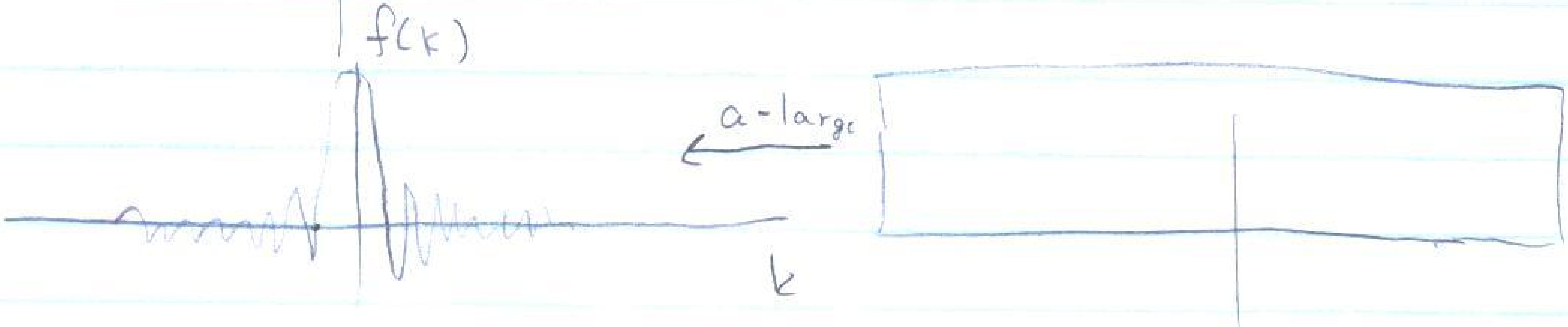
$$f(x) = L \int \frac{dk}{2\pi} \frac{e^{+ik\cdot x}}{\sqrt{L}} \frac{f_{cont}(k)}{\sqrt{L}}$$

$$\boxed{f(x) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{+ik\cdot x} f_{cont}(k)}$$

We will
normally
work wi
Box nota

$$f(k) = \frac{1}{\sqrt{L}} \frac{\sin(ka/2)}{ka/2}$$





$\Delta k \quad \Delta x \sim 1$

$p = \frac{h}{\lambda} = \frac{h}{2\pi} \frac{2\pi}{\lambda} = \hbar k$

$\hbar \Delta k \quad \Delta x \sim \hbar$

$\Delta p \ \Delta x \sim \hbar$

Summary k-space is momentum space:

$$p = \hbar k$$

$$\phi_p = \frac{1}{\sqrt{L}} e^{+ip\frac{x}{\hbar}}$$

$$\sum_p = \int L \frac{dp}{2\pi\hbar}$$

$$\psi(x,t) = \sum_p \frac{e^{+ip\cdot x}}{\sqrt{L}} f(p)$$

$$f(p) = \int_L \frac{e^{-ipx}}{\sqrt{L}} f(x) \, dx$$

For a free partice momentum eigenstates are energy eigenstates

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \cancel{V}^{0} \right] \phi_p(x) = E_p \, \phi_p(x)$$

$$\frac{\partial}{\partial x} \frac{1}{\sqrt{L}} e^{ipx/\hbar} = +\frac{ip}{\hbar} \frac{1}{\sqrt{L}} e^{+ip\frac{x}{\hbar}}$$

$$\frac{\partial^2}{\partial x^2} \frac{1}{\sqrt{L}} e^{+ip\cdot x} = -\frac{p^2}{\hbar^2} \frac{1}{\sqrt{L}} e^{+ip\cdot x}$$

$$\underbrace{\hat{H}_{free}}_{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}} \underbrace{\phi_p^{(x)}}_{\frac{e^{+ip\cdot x}}{\sqrt{L}}} = \underbrace{E_p}_{\frac{p^2}{2m}} \underbrace{\phi_p(x)}_{\frac{e^{+ip\cdot x}}{\sqrt{L}}} \checkmark$$

Now every thing we said in one - D also applies to 3D

$$P_x = -i\hbar \frac{\partial}{\partial x} \qquad P_y = -i\hbar \frac{\partial}{\partial p_y} \qquad P_z = -i\hbar \frac{\partial}{\partial z}$$

- $$\phi_{\vec{p}} = \frac{1}{\sqrt{V}} e^{+i\vec{p}\cdot x}$$

- $$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \phi_{\vec{p}}(x) = \frac{p^2}{2m} \phi_{\vec{p}}(x)$$

$$\psi(p) = \int_V \frac{e^{-i\vec{p}\cdot\vec{x}/\hbar}}{\sqrt{V}} \psi(x)$$

$$\psi(x) = \sum_p \frac{e^{+i\vec{p}\cdot\vec{x}/\hbar}}{\sqrt{V}} \psi(\vec{p})$$

$$\downarrow$$

$$\psi(x) = \int V \frac{d^3p}{(2\pi)^3} \frac{e^{+ip\cdot x}}{\sqrt{V}} \psi(p)$$

Last Times

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