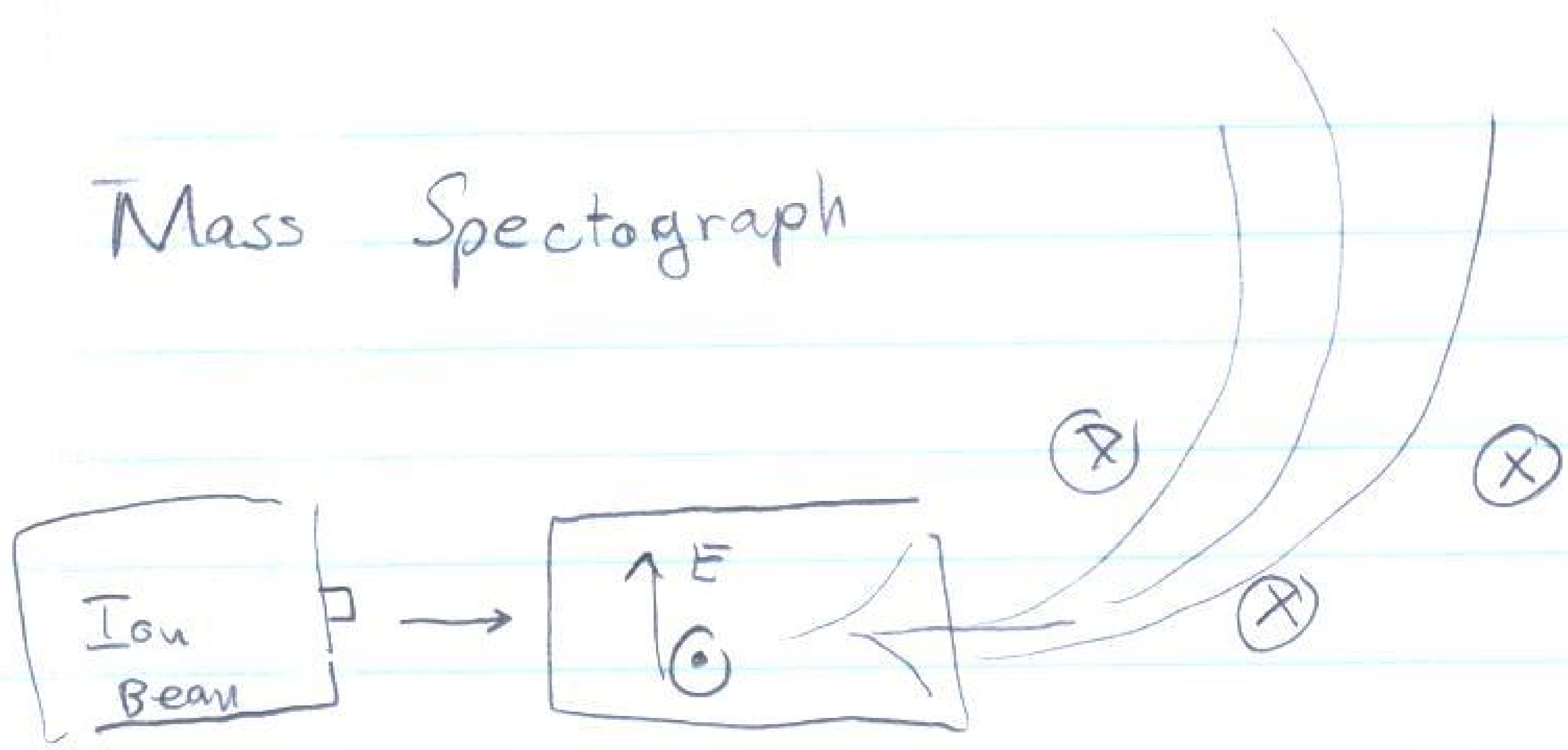
Mass Spectograph



$F = qE$ up $\qquad F = qvB$ down

Straight Line:

$$\cancel{q}E = \cancel{q}vB$$

$$\frac{E}{B} = v$$

Now we enter a magnetic Field:

$$\frac{mv^2}{R} = qBv \qquad R = \frac{mv}{qB}$$

$$\boxed{m = \frac{qRB^2}{E}}$$

Doublet method

Compare Two species @ same mass

C_9H_{20} and $C_{10}H_8$

$$\Delta = [m(C_9H_{20}) - m(C_{10}H_8)]$$

$$\Delta = 12\,m(H) - m(^{12}C)$$ ← measured precisely

$$m(H) = \frac{1}{12}[m(^{12}C) + \Delta]$$

$$m(H) = 1.00782503 \pm .00000001$$

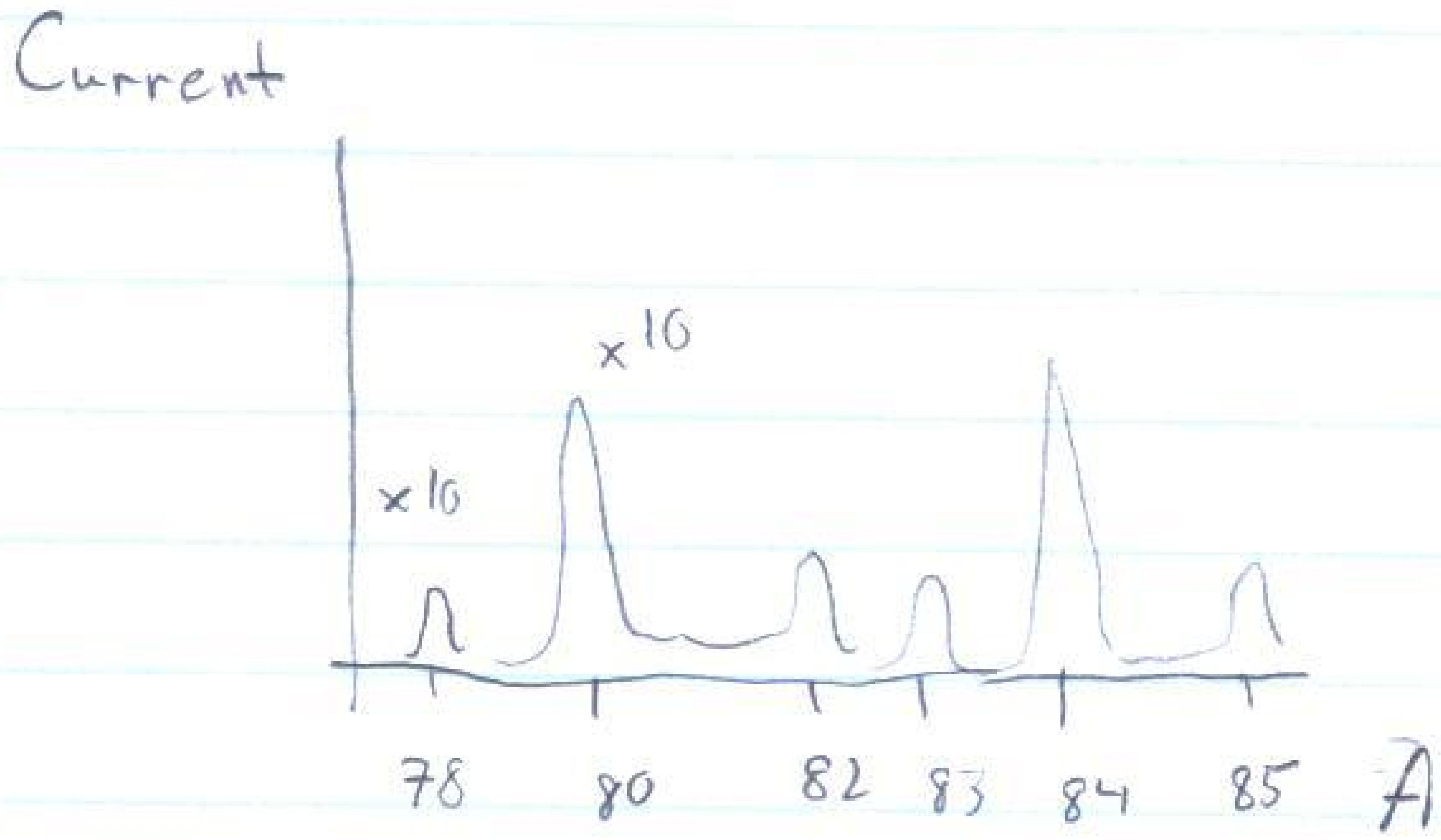
Then to determine say ^{14}N look at C_2H_4 and N_2

$$\Delta' = m(C_2H_4) - 2m(^{14}N)$$ ← measured precisely

$$m(^{14}N) = m(^{12}C) + 2m(^1H) - \frac{1}{2}\Delta$$

$$= 14.00307396 \pm 0.00.0000002$$

The spectro-graph can also be used to measure the relative abundance



$^{78}Kr = 0.356\%$

$^{80}Kr = 2.27\%$

$^{82}Kr = 11.6\%$

$^{83}Kr = 11.5\%$

$^{84}Kr = 57\%$

$^{86}Kr = 17.3\%$

Determining masses of Unstable Nuclei

$$x + X \to y + Y$$

$$Q = [m(x) + M(X) - m(y) - M(Y)]\, c^2$$

Measure the kinetic energy of outgoing constituents, and incoming energies

Example:

$$^{1}H + {}^{14}N \to {}^{12}N + {}^{3}H$$

Measure the kinetic energies $Q = -22$ MeV

$$m(^{12}N) = m(^{1}H) + m(^{14}N) - m(^{3}H) - Q$$

Now we can fit the masses

$$B(A,Z) = \underset{\substack{\uparrow \\ \text{Volume } \frac{B}{A} \text{ Const}}}{a_V A} - a_s A^{2/3} - \underset{\substack{\uparrow \\ 0.72\,\text{MeV}}}{a_c} \frac{Z(Z-1)}{A^{1/3}} - \underset{\substack{\uparrow \\ 34\,\text{MeV}}}{a_{sym}} \frac{A(1-2Z)^2}{A}$$

Then there is a coulomb repulsion:

$$V = \frac{k_e Q}{R^2}\left(1 - \frac{1}{2}\frac{r^2}{R}\right)$$

$$U_{\substack{electro\\stat}} = \int 4\pi r^2 \, dr \, \rho \, \frac{k_e q(r)}{r}$$

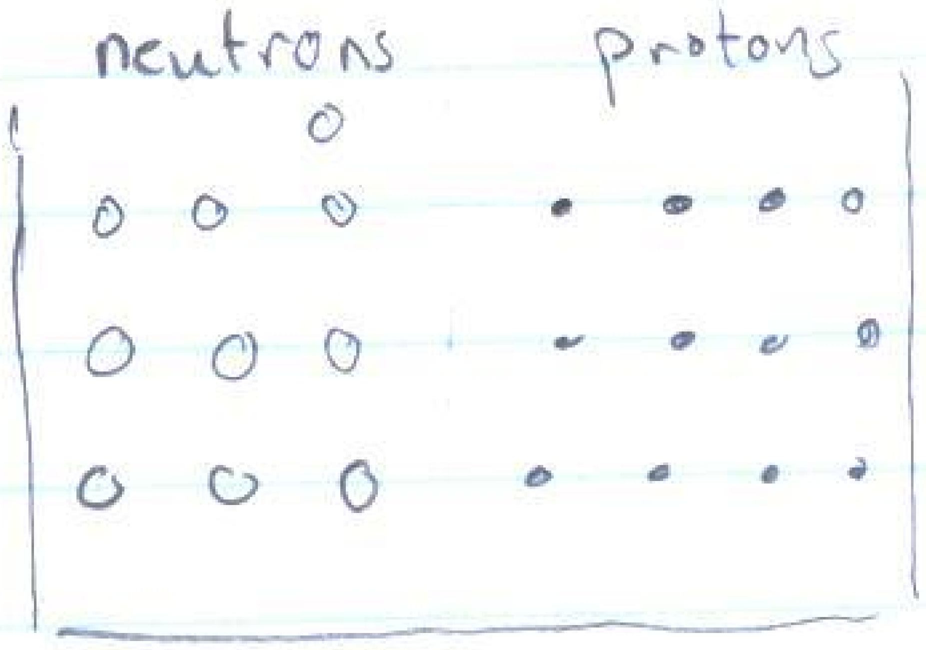
$$= 4\pi k_e \rho \int_0^R \frac{4}{3}\pi r^4 \rho \, dr$$

$$= 4\pi k_e \frac{Q^2}{\left(\frac{4}{3}\pi R^3\right)^2} \frac{4\pi}{3} \frac{R^5}{5} = \frac{3}{5} \frac{k_e Q^2}{R}$$

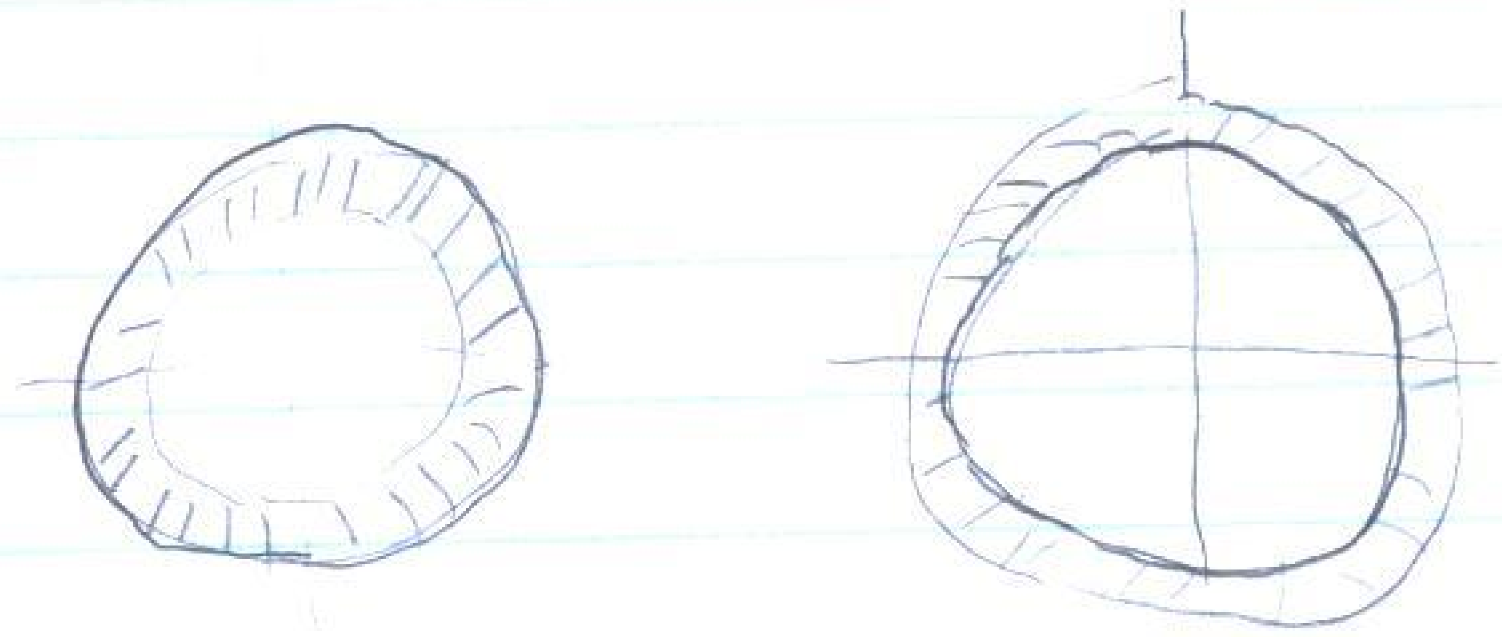
$$U_{\substack{elec\\stat}} = \frac{3}{5} \frac{k_e e^2}{\hbar c} \cdot \frac{\hbar c}{R_A}$$

$$= \frac{3}{5} \underbrace{\alpha_1}_{\frac{1}{137}} \cdot \frac{197 \text{ MeV fm}}{1.2\text{fm } A^{1/3}} = 0.72 \text{ MeV}$$

Then consider



If we convert a proton to a neutron then we will add to the fermi see



$$N = \left(\frac{g}{2\pi^2}\right)\frac{1}{3}\left(\frac{p_f}{\hbar}\right)^3 \cdot V \qquad E = \frac{g}{2\pi^2}\,\frac{1}{3}\left(\frac{p_f}{\hbar}\right)^3\left(\frac{3}{5}\frac{p_f^2}{2m}\right)\cdot V$$

$$\left(\frac{6\pi^2}{g}\frac{N}{V}\right)^{1/3} = \frac{p_f}{\hbar}$$

$$E = V\, C_E\, p_f^5 \qquad N = V\, C_N\, p_f^3$$

Then

$$\Delta E_{TOT} = \Delta E_p + \Delta E_N = 2\left[\overbrace{\frac{1}{2}\frac{\partial^2 E}{\partial p_f^2}(\Delta p_F)^2}^{\Delta E_p} + \overbrace{\frac{1}{2}\frac{\partial^2 E}{\partial p_f^2}(\Delta p_f)^2}^{\Delta E_N}\right]$$

$$= \frac{\partial^2 E}{\partial p_f^2}(\Delta p_f)^2$$

$$E = V \cdot C_E\, p_f^5 \qquad C_E = \frac{g}{2\pi^2}\,\frac{1}{5}\cdot\frac{1}{2m}\,\frac{1}{\hbar^3}$$

$$\frac{\partial^2 E}{\partial p_f^2} = 5 \cdot 4 \cdot V C_E\, p_f^3$$

$$\Delta N = \frac{\partial N}{\partial p_f}\Delta p_f = 3C_N \cdot V\, p_f^2\, \Delta p_f$$

$$\Delta E_{TOT} = 20\, C_E \cdot V\, p_f^3 \cdot \left[\frac{\Delta N}{(3C_N \cdot V \cdot p_f^2)}\right]^2$$

$$\Delta E_{TOT} = \frac{20\, C_E \cdot \cancel{V}}{9\, C_N^2\, V^{\cancel{2}}}\,\frac{1}{p_f}\,(\Delta N)^2 \qquad p_f =$$

$$\Delta E_{TOT} = \frac{20}{9}\,\frac{C_E}{C_N^2}\,\frac{1}{(p_f)}\,\frac{(\Delta N)^2}{V}$$

Const

$$\Delta N = \left(\frac{N-Z}{2}\right)$$

$$V = \frac{A}{\rho_0}$$

So - $$\Delta E_{TOT} = \frac{20}{9} \frac{C_E}{C_N^2} \frac{1}{p_f} \cdot \frac{1}{4} \cdot \frac{(N-Z)^2}{\frac{A}{\rho_0}}$$

$$C_N p_f^3 = \frac{\rho_0}{2} \longrightarrow$$

$$\Delta E_{TOT} = \frac{20}{9} \frac{C_E}{C_N^2} \cdot \frac{1}{p_f} \cdot \frac{1}{4} \overbrace{\rho_0}^{2C_N p_f^3} \frac{(N-Z)^2}{A}$$

$$\Delta E_{TOT} = \frac{10}{9} \frac{C_E}{C_N} p_f^2 \left(\frac{N-Z}{A}\right)^2$$

$$\Delta E_{TOT} = \frac{10}{9} \underbrace{\frac{p_f^2}{2m}}_{\sim 25\,\text{MeV from before}} \left(\frac{N-Z}{A}\right)^2$$

$$\Delta E \sim 27\ \text{MeV} \left(\frac{N-Z}{A}\right)^2$$

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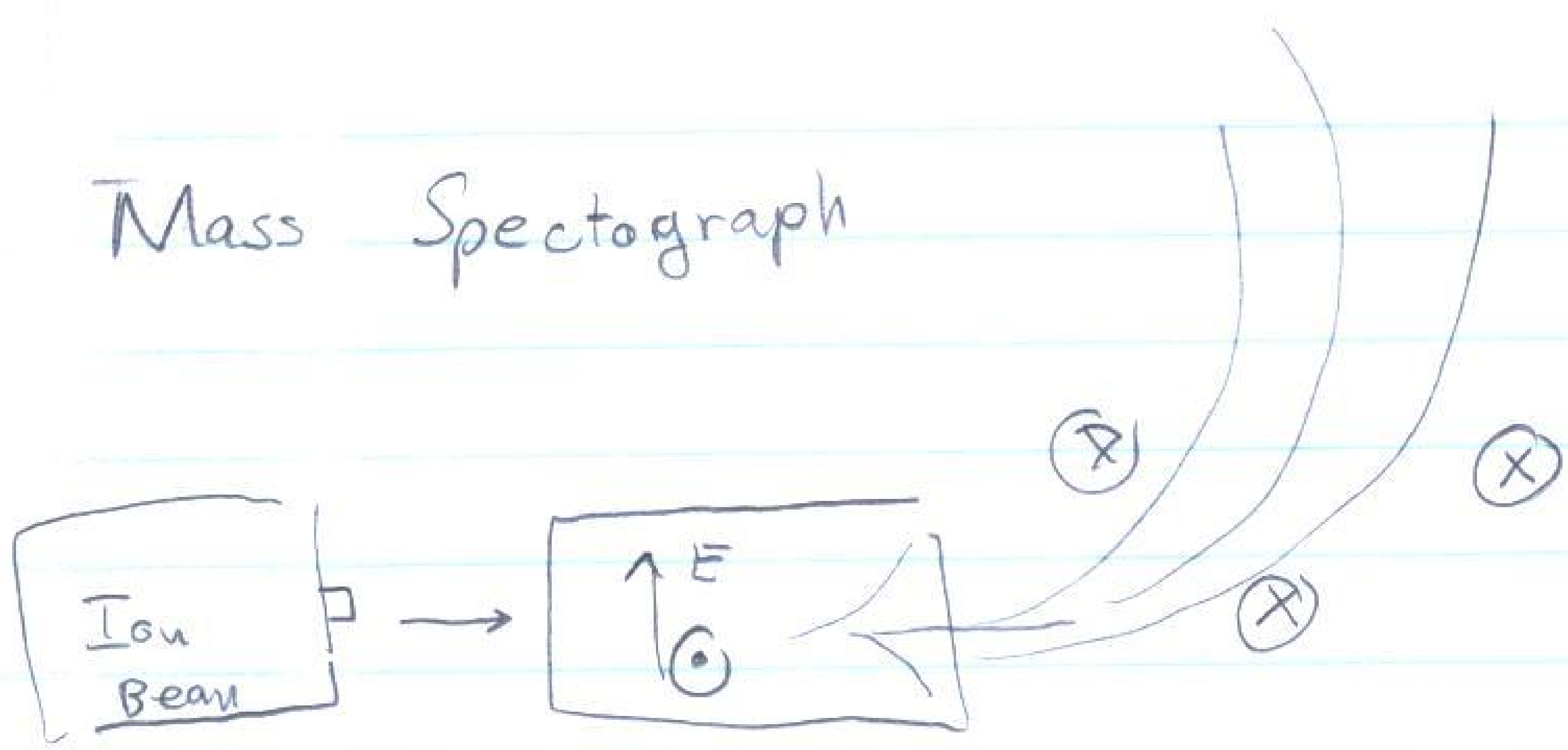

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