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Stability

- Only Four Odd-Odd Nuclei (^2H, ^6Li, ^{10}B, ^{14}
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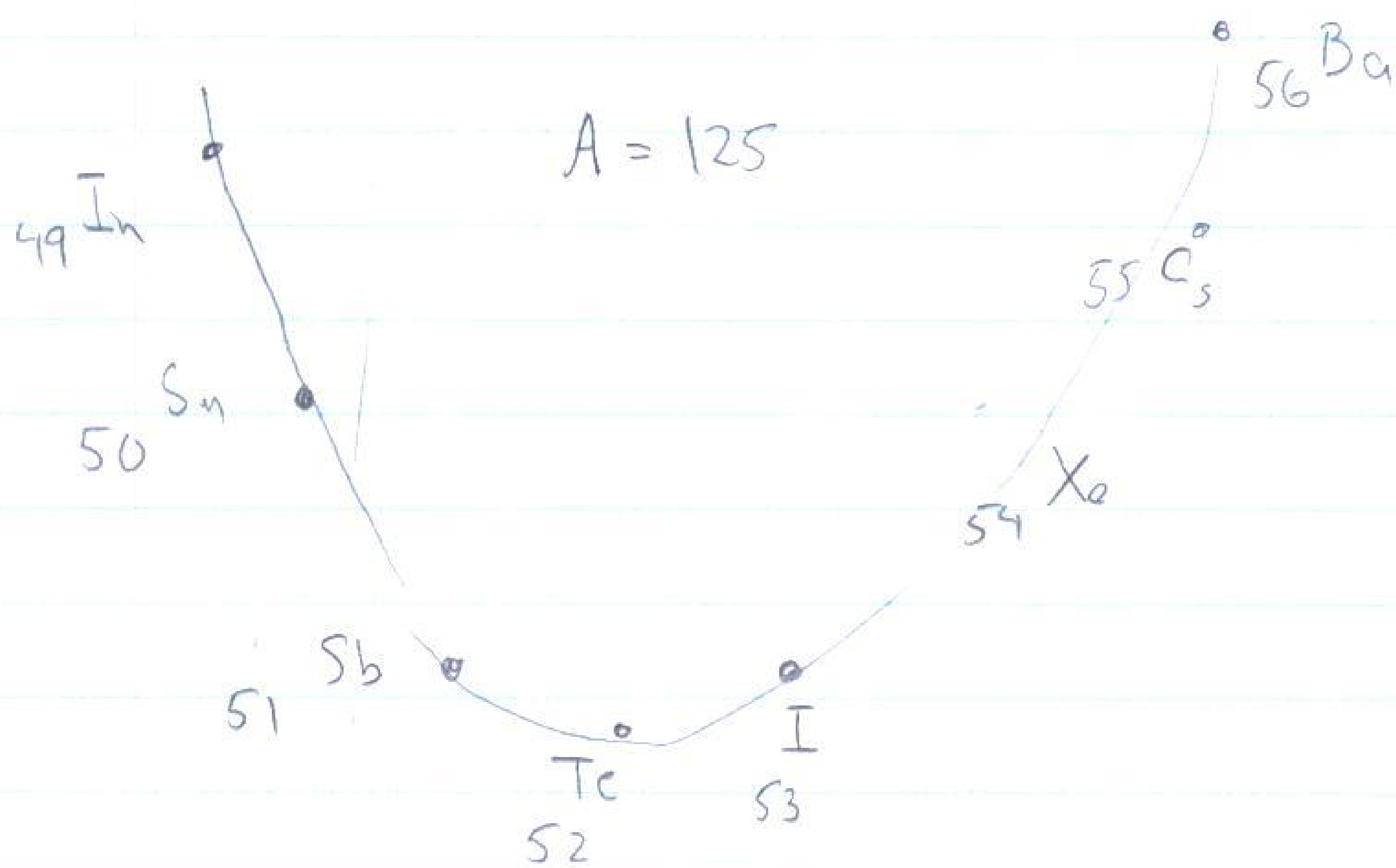
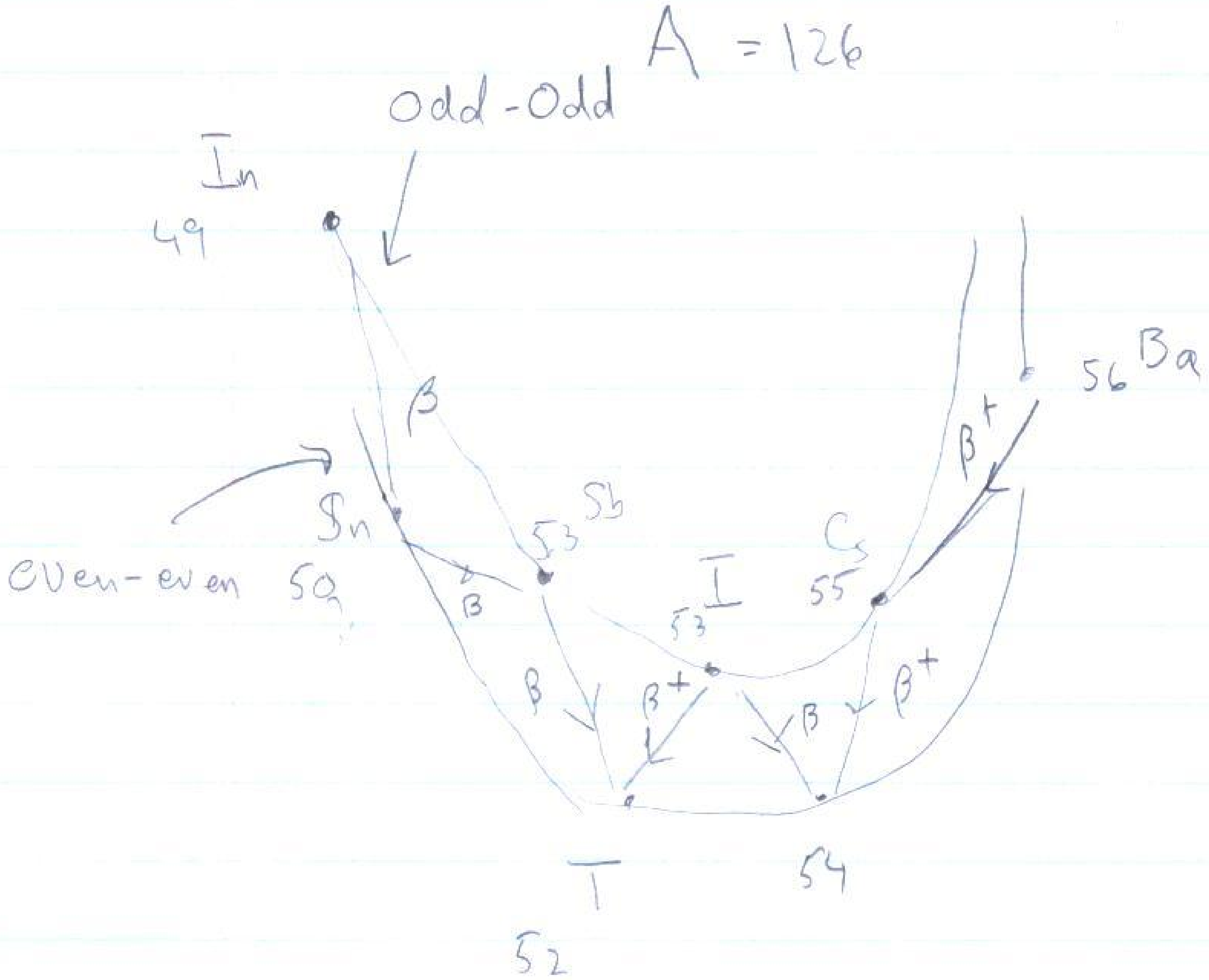
Decay Chains





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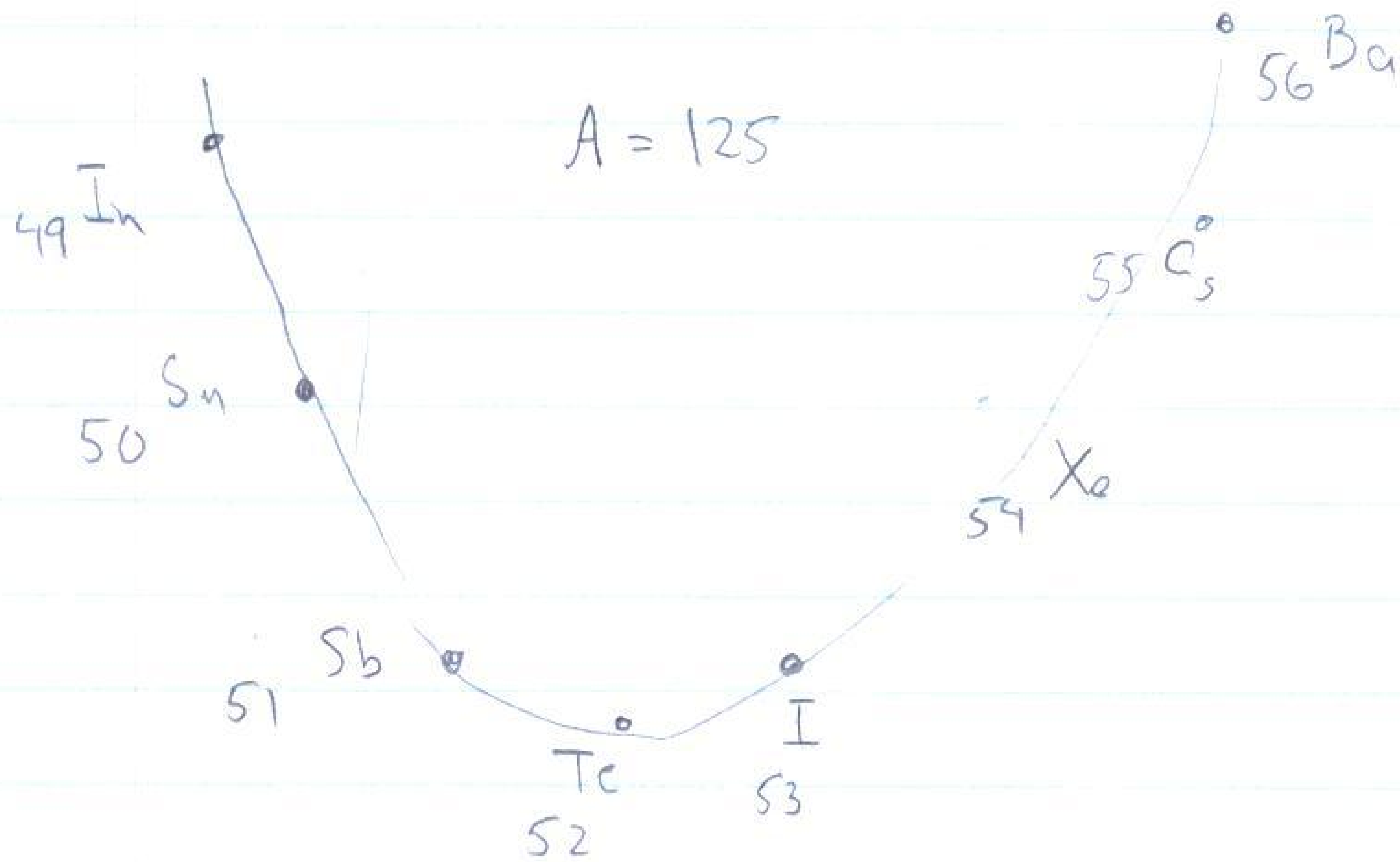
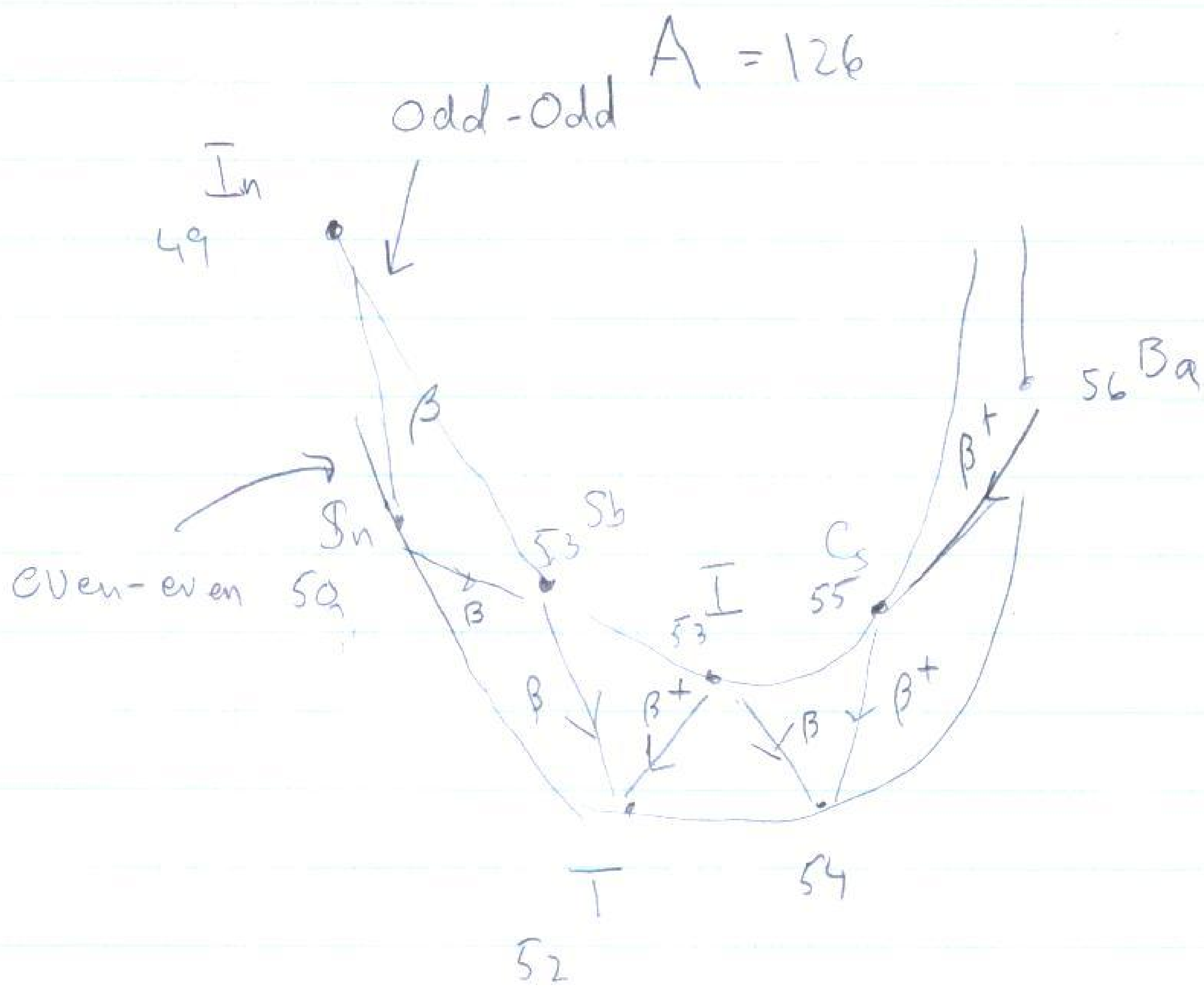
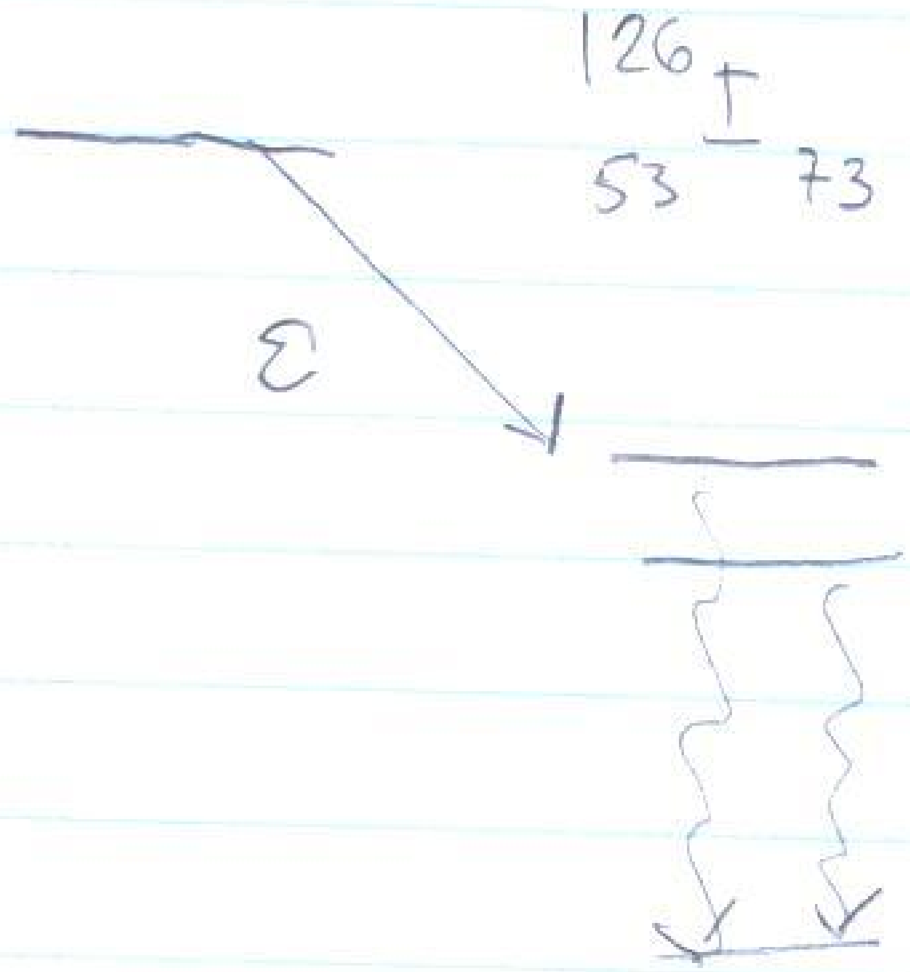




Now once you decay then what



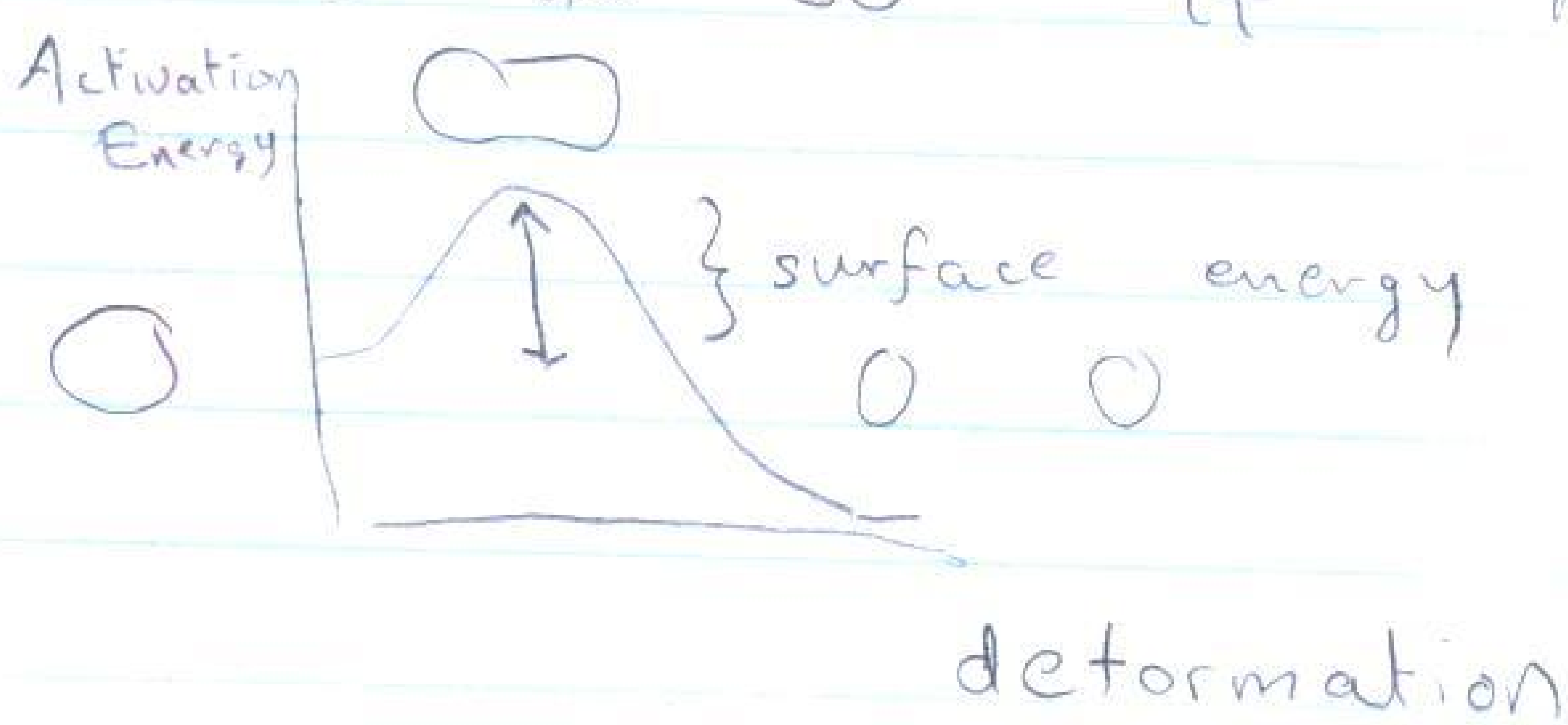
- Usually land up in some excited state and then emit γ's

α decay

For very large nuclei the coulomb repulsion begins to be important

- The nucleus can reduce its coulomb repulsion by splitting into two

- To do so it must deform



Consider an ellipsoid

$$V = \frac{4\pi}{3} a b^2 \qquad a = R(1+\varepsilon)$$

$$b = \frac{R}{\sqrt{1+\varepsilon}}$$

Then, Area $= 4\pi R^2 \left(1 + \frac{2}{5}\varepsilon^2\right)$:

$\Delta E = \frac{2}{5} a_s A^{2/3} \varepsilon^2$ ← energy increases for more surface area

Coulomb Repulsion is less:

$E_{\text{coulomb}} \propto 1 - \frac{2}{5}\varepsilon^2 \qquad \Delta E_{\text{coulomb}} = -\frac{1}{5} a_c Z^2 \frac{e^2}{A^{1/3}}$

Putting These pieces together

$$\Delta E = -\Delta B = \left(\overset{①}{\frac{2}{5} a_s A^{2/3} \varepsilon^2} - \overset{②}{\frac{1}{5} a_c \frac{Z^2 \varepsilon^2}{A^{1/3}}} \right)$$

As the deformation

A increase until we reach

$$\frac{2}{5} a_s A^{2/3} \varepsilon^2 = \frac{1}{5} a_c \frac{Z^2 \varepsilon^2}{A^{1/3}}$$

$$\frac{2a_s}{a_c} \simeq \frac{Z^2}{A} \simeq 49$$

At his point the nucleus becomes unstable and break apart

Nulear Drip Line

- As the number of protons and neutrons increases eventually it becomes more favorale to split into a proton nuleaus

Mass Formula

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{A} + \delta$$

$a_V = 15$ MeV $\quad a_s = 13$ MeV

$a_c \simeq 0.6$ MeV

$a_{sym} = 19$ MeV

$\delta = -33.5$ MeV

Deuteron

- n and proton ~ heavy hydrogen ${}^2_1H_1$
- mass determined by the mass doublet method

deuterium

measure: $\Delta = m(C_6 H_{12}) - m(C_6 D_6)$

$$m({}^2_1H_1) = 2.0141017\,8 \pm 21 \text{ u}$$

$$B({}^2_1H_1) = 2.22463 \pm .00004 \text{ MeV}$$

Can also measure:

$$^{1}H + n \rightarrow {}^{2}H + \gamma$$

$$\gamma + {}^{2}H \rightarrow {}^{1}H + n$$

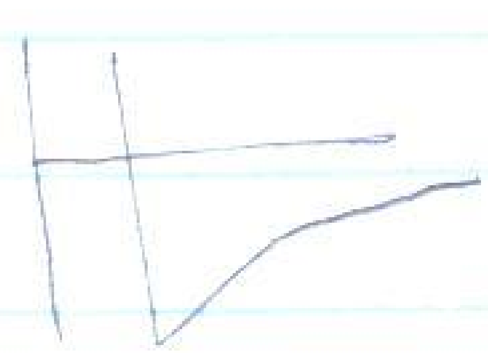
And by meausing the energies of the outgoing products deduce something comparables

$$B({}^{2}_{1}H_{1}) = 2.2 \text{ MeV}$$

- Somewhat less bound than typical nuclei

Also compare BE with Kinetic energy:

$$KE \sim \frac{p^2}{2m} \sim \frac{\hbar^2}{2m(\Delta x)^2}$$



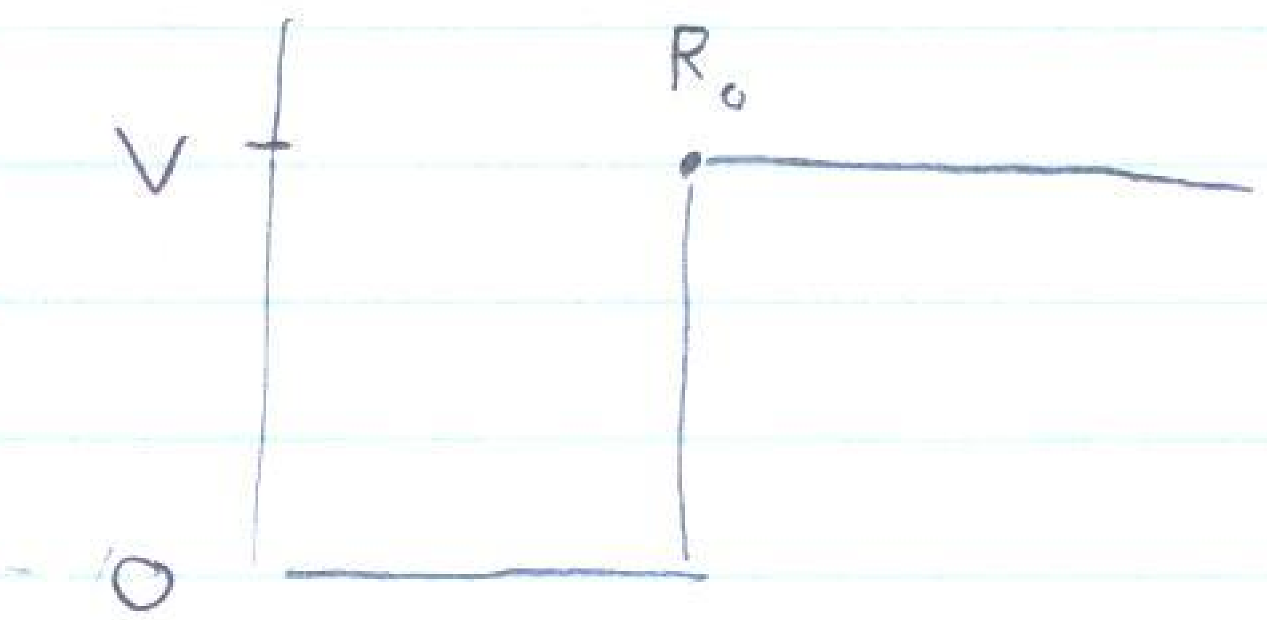
$$KE \sim \frac{(\hbar c)^2}{2mc^2} \frac{1}{(\Delta x)^2}$$

$$KE \sim \frac{(0.2 \text{ GeV fm})^2}{0.938 \text{ GeV fm}} \frac{1}{(1.0 \text{ fm})^2} \sim 40 \text{ MeV}$$

$$KE \gg BE$$

The deuteron is the result of a very precise cancellation of Kinetic and Potential energies

Simple Model For the Deuteron



$$\left[\frac{-\hbar}{2m}\frac{d^2\phi_E}{dr^2} + V(r)\right]\phi_E = E\phi_E$$

$$\phi_E \begin{cases} \sin(kx) & x < R \qquad k = \sqrt{\frac{2m}{\hbar^2}E} \\ Be^{-\bar{K}x} & x > R \qquad \bar{K} = \sqrt{\frac{2m}{\hbar^2}(V-E)} \end{cases}$$

Continuity: $\sin(kR) = Be^{-\bar{K}R}$

$$k\cos(kR) = -B\bar{K}e^{-\bar{K}R}$$

$$k\cos(kR) = -\bar{K}\sin(kR)$$

S_0

$$\frac{k}{\bar{K}} = -\tan(kR)$$

with

$$\bar{K}^2 = \frac{2m(V-E)}{\hbar^2}$$

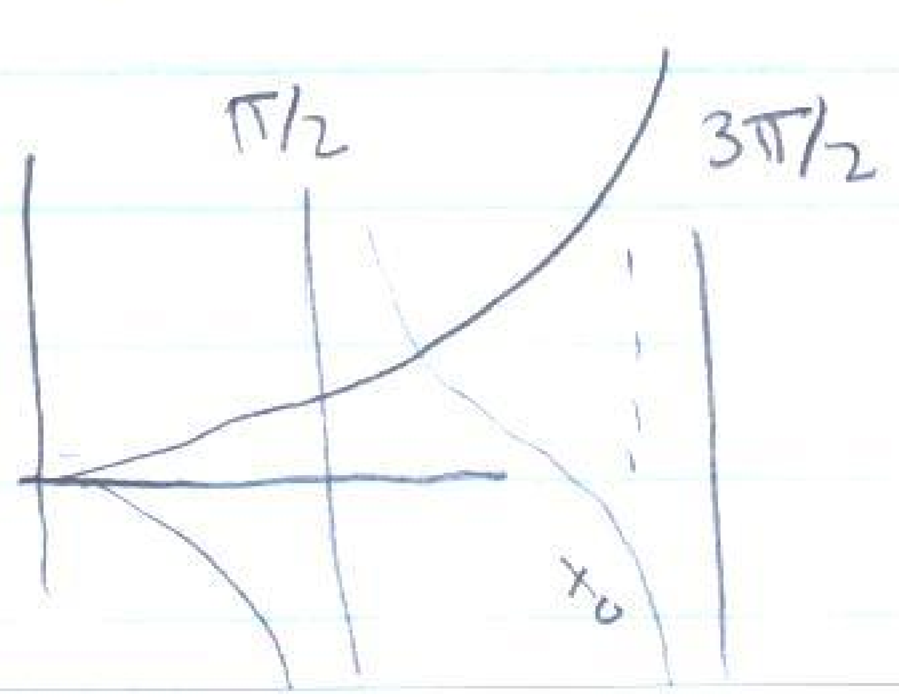
$$= \frac{2mV}{\hbar^2} - k^2$$

$$\bar{K}^2 = k_D^2 - k^2 \qquad \text{with} \quad \boxed{k_D^2 \equiv \frac{2m}{\hbar^2} V}$$

$$\frac{k}{\sqrt{k_D^2 - k^2}} = -\tan(kR)$$

$$\frac{kR}{\sqrt{(k_D R)^2 - (kR)^2}} = -\tan kR$$

$$\frac{x}{\sqrt{x_0^2 - x^2}} = -\tan x \qquad \boxed{\begin{aligned} x &\equiv kR \\ x_0 &\equiv k_D R \end{aligned}}$$



Min

$x_0 = \frac{\pi}{2}$ minimum potential depth

$$\sqrt{\frac{2mR^2 V}{\hbar^2}} = \frac{\pi}{2}$$

$$\boxed{V_{min} = \frac{\hbar^2}{2mR^2}\left(\frac{\pi}{2}\right)^2}$$

$x_0 = \frac{3\pi}{2}$ maximum potential depth

$$\frac{\hbar^2}{2mR^2} \simeq 20 \text{ MeV}$$

$$V_{min} \simeq \left(\frac{\pi}{2}\right)^2 \frac{\hbar^2}{2mR^2} = 50 \text{ MeV}$$

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