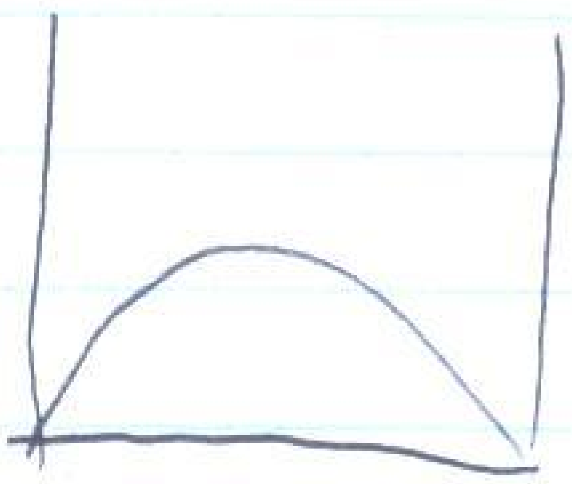
Two-particles in the box



$$\phi_k(x) = \sqrt{\frac{2}{L}}\sin kx \qquad n = 1, 2, 3, \ldots$$

$$k = \frac{\pi n}{L}$$

$$E_k = \frac{\hbar^2 k^2}{2m}$$

- $H\,\psi(x_1, x_2)$

- $H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} + \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2}$

$$\hat{H}\,\phi_E(x_1, x_2) = E\,\phi_E(x_1, x_2)$$

$$\left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} + \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2}\right)\phi_E(x_1, x_2) = E\,\phi_E(x_1, x_2)$$

Try: $\phi_{kk'} = \phi_k(x_1)\,\phi_{k'}(x_2) = \phi_{E_{kk'}}$

Works:

$$E_{kk'} = \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2 k'^2}{2m} = E_k + E_{k'}$$

To bad it doesn't really for identical

Need more

$$\psi(x_1, x_2) = \psi(x_2, x_1) \qquad \text{(Bosons)}$$

But quantum mechanics doesn't care about sign

$$\psi(x_1, x_2) = -\psi(x_2, x_1) \qquad \text{(Fermions)}$$

electrons, nucleons

So natural guess:

$$\phi_E(x_1 x_2) = \begin{cases} [\phi_k(x_1)\,\phi_{k'}(x_2) + \phi_k(x_2)\,\phi_{k'}(x_1)] & \text{Bosons} \\ [\phi_k(x_1)\,\phi_{k'}(x_2) - \phi_k(x_2)\,\phi_{k'}(x_1)] & \text{Fermions} \end{cases}$$

$$= \begin{vmatrix} \phi_k(x_1) & \phi_{k'}(x_1) \\ \phi_k(x_2) & \phi_{k'}(x_2) \end{vmatrix}$$

Norm

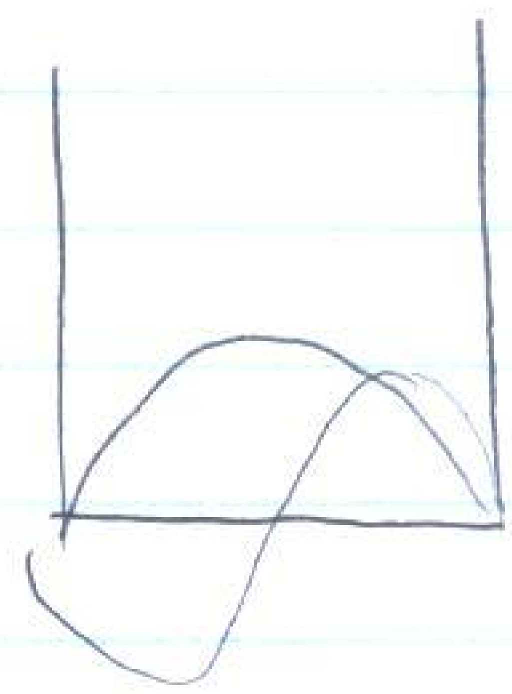
$$\int \frac{dx_1\, dx_2}{2} \,|\, \phi_E(x_1 x_2) \,|^2 = 1$$

$$\int \frac{dx_1\, dx_2}{2} \left[\phi_k^*(x_1)\,\phi_{k'}^*(x_2) - \phi_k^*(x_1)\,\phi_{k'}^*(x_2)\right]$$

$$\left[\phi_k(x_1)\,\phi_{k'}(x_2) - \phi_k(x_2)\,\phi_{k'}(x_1)\right] = 1 \checkmark$$

Consequences

$$\phi_{kk'}(x_1, x_2) = \phi_k(x_1)\,\phi_{k'}(x_2) - \phi_k(x_2)\,\phi_{k'}(x_1)$$



if $k = k'$ the wave function vanishes

So the lowest eigenstate

- has one partice in lowest mode
- one partice in the next mode etc

Three particles:

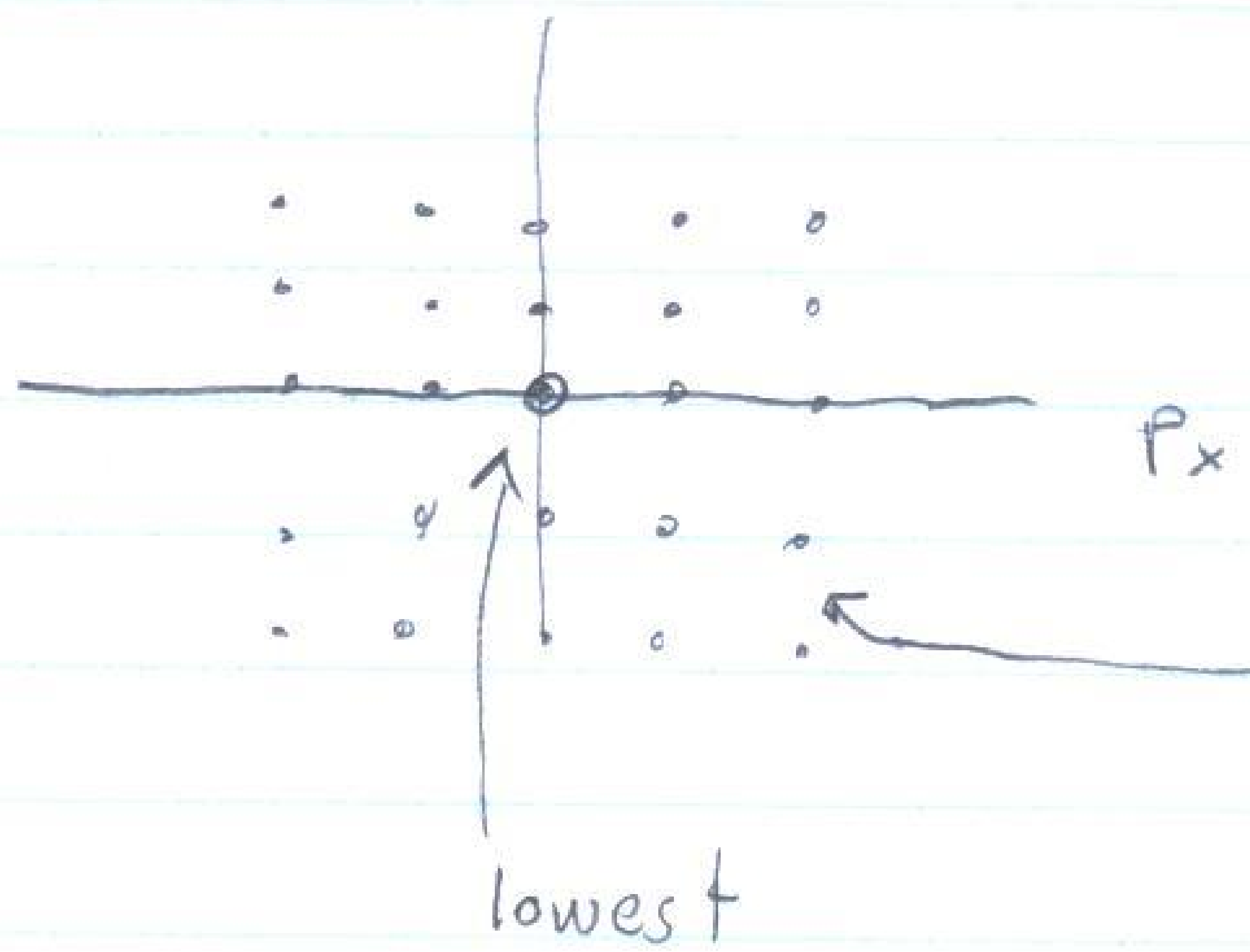
$$\phi_{kk'k''}(x_1 x_2 x_3) = \begin{vmatrix} \phi_k(x_1) & \phi_{k'} & \phi_{k''} \\ \phi_k(x_2) & \phi_{k'} & \phi_{k''} \\ \phi_k(x_3) & \phi_{k'} & \phi_{k''} \end{vmatrix}$$

If any momenta are the same wave fcn vanishes

Lowes Eigen-state

- one in ground
- one in next
- one in next

Then the energy eigenstates are for a free partice



$$p_x = \frac{2\pi n_x}{L}$$

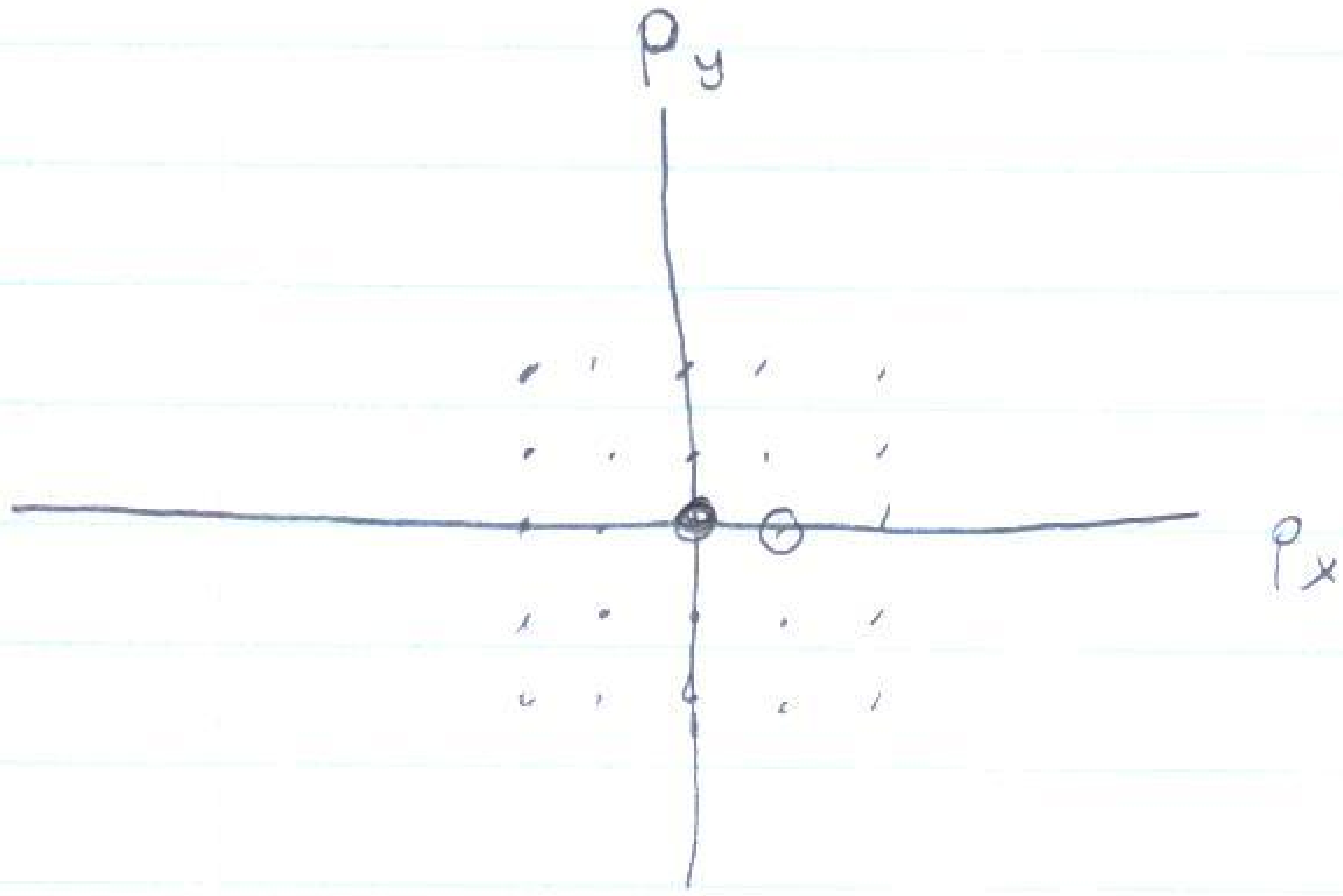
$$p_y = \frac{2\pi n_y}{L}$$

every point labels an energy eigenstate

Now suppose we have two particles

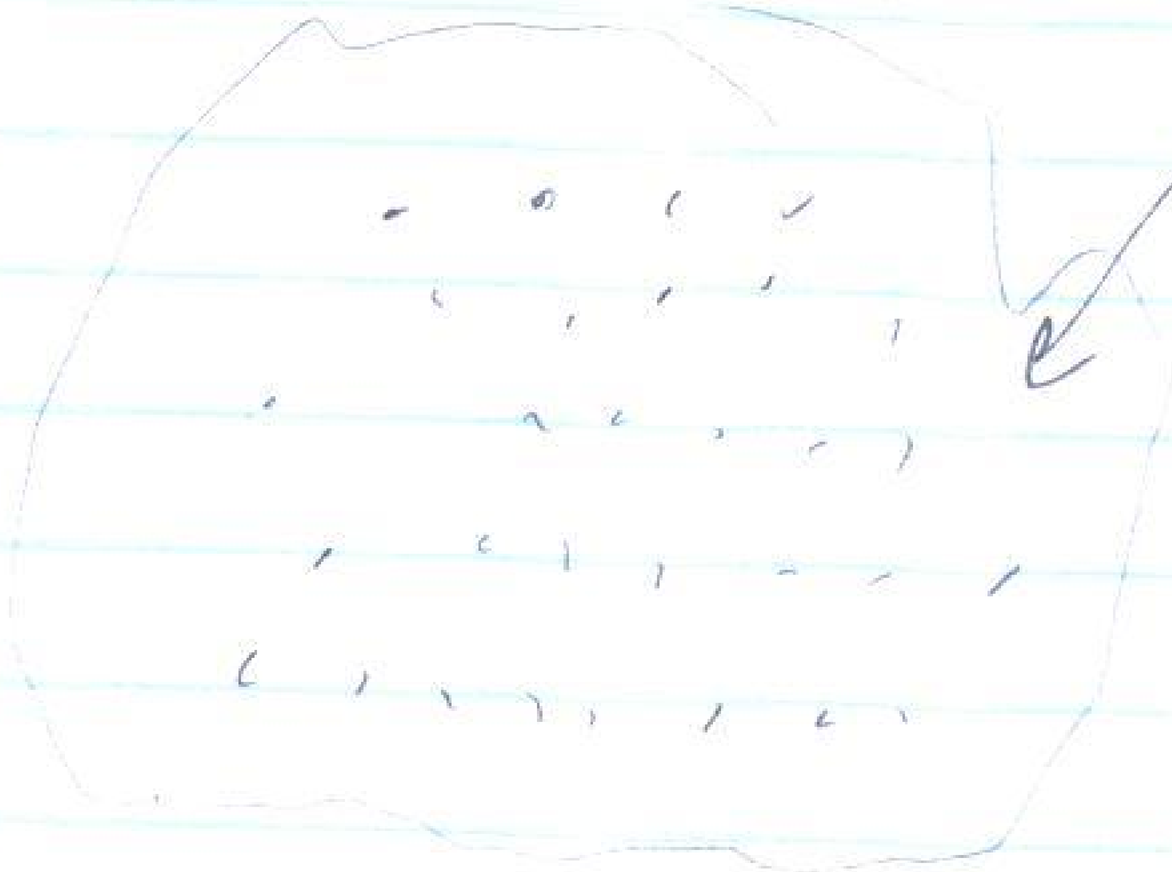
$$\psi_{pp'} = \frac{1}{\sqrt{2}}\left[\phi_{\vec{p}}(x_1)\,\phi_{\vec{p}'}(x_2) - \phi_{\vec{p}}(x_2)\,\phi_{\vec{p}'}(x_1)\right]$$

→ vanishes if $p = p'$ p & p' must be differen

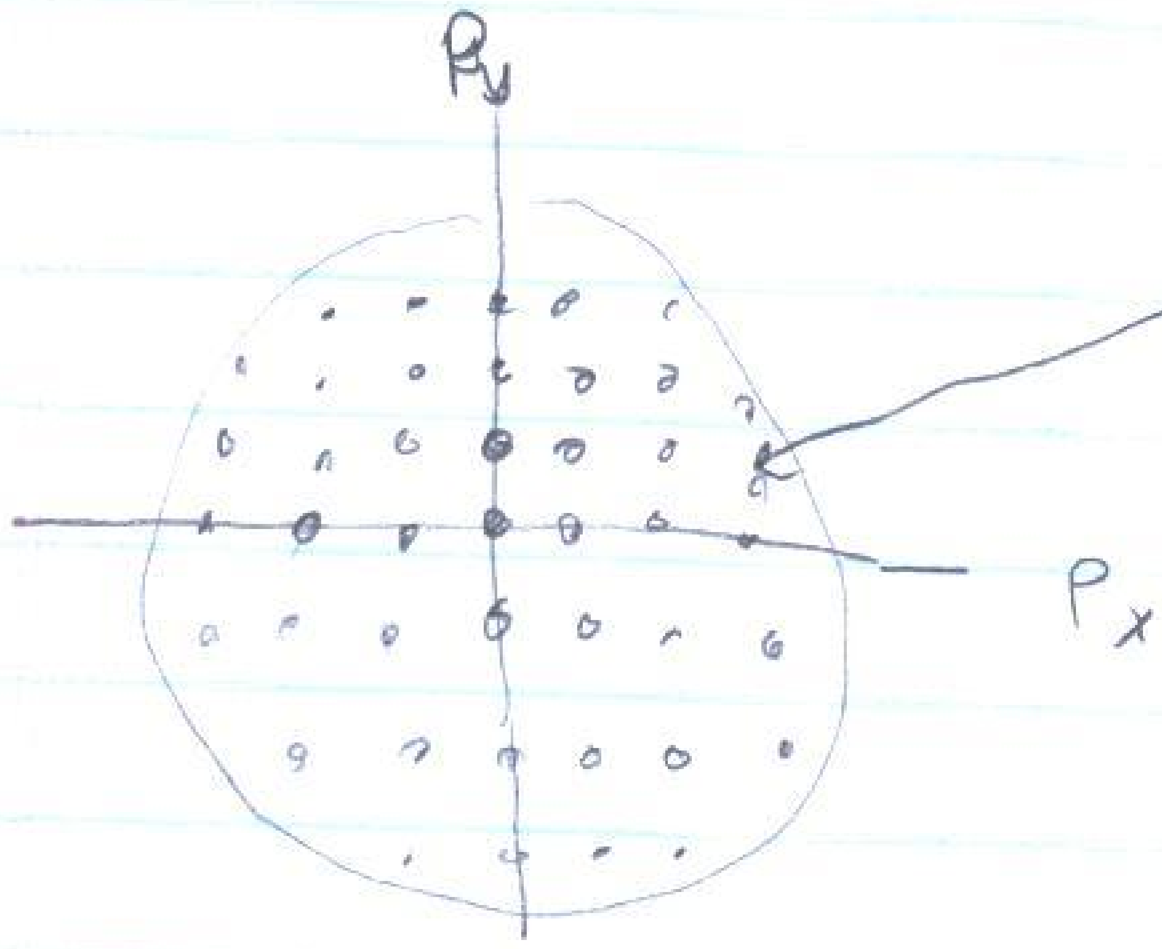


← lowest eigenstate with two particles in the box

Now consider nuclear matter



Size V, # N



Fill all states up to p_f

$$N = \sum_{p}^{p_f} = \int \frac{V\, d^3p}{(2\pi\hbar)^3} = \frac{V}{(2\pi)^3}\,\frac{4\pi}{3}\,p_f^3$$

$$\frac{N}{V} = \frac{1}{2\pi^2}\,\frac{p_f^3}{3\hbar^3}$$

Estimate of Fermi momentum for nuclear matter

$$\rho_0 = \frac{N}{V} = \frac{1}{6}\ \frac{\text{nucleons}}{fm^3} = \frac{1}{6\pi^2}\left(\frac{p_f}{\hbar}\right)^3$$

$$\frac{p_f}{\hbar} = (6\pi^2\rho_0)^{1/3} = 2.14\ \frac{1}{fm}$$

$$c p_f = 2.14\ \frac{1}{fm}\cdot \underbrace{\hbar c}_{200\ MeV\,fm}$$

$$c p_f \approx 400\ MeV$$

quite large — doesn't include spin-iso-spin

with this ≈ 200 MeV

$$\frac{BE}{A} \simeq 8\ \frac{MeV}{A}$$

Problem Compute The average Energy

$$\langle E\rangle = \sum_{p}^{p_f} \frac{p^2}{2m} = \int^{p_f} \frac{V d^3p}{(2\pi)^3\hbar^3}\ \frac{p^2}{2m}$$

$$= \frac{V}{2\pi^2}\int^{p} p^2\,dp\ \frac{p^2}{2m} = \frac{V}{2\pi^2}\ \frac{1}{2m}\ \frac{1}{\hbar^3}\ \frac{p_f^5}{5}$$

$$\frac{\langle E\rangle}{\langle N\rangle} = \frac{3}{5}\ \frac{p_f^2}{2m}$$

$$\frac{\langle E \rangle}{\langle N \rangle} \simeq 51.1 \text{ MeV}$$

much more compatible

$$BE \simeq 8 \frac{\text{MeV}}{A}$$

with spin iso-spin

$\simeq 25$ MeV

Last Time:

$$f(x) = \sum_{k} \phi_k(x)\, f(k) \qquad \phi_k = \frac{e^{+ik\cdot x}}{\sqrt{L}}$$

$$k = \frac{2\pi n}{L}, \quad n = -\infty, \ldots$$

$$f(k) = \int_{-L}^{L} \frac{e^{-ikx}}{\sqrt{L}} f(x)$$

This follows because $\int_L \phi_k^* \phi_{k'} = \delta_{kk'}$

k

$\phi = \frac{1}{\sqrt{}}$