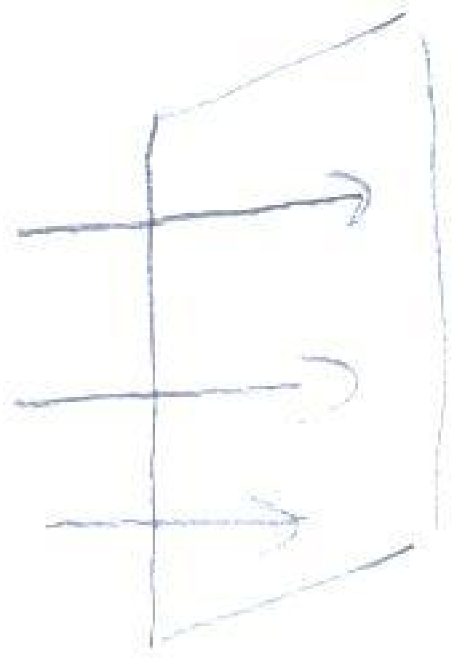
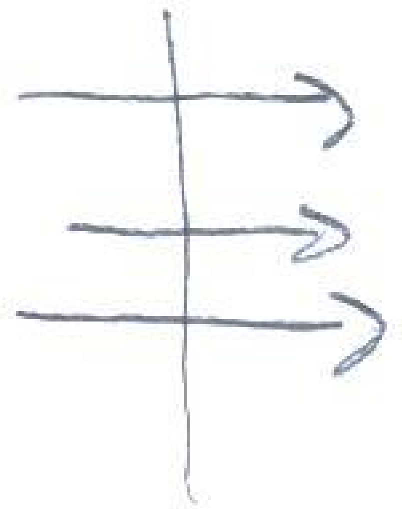
Electric Flux

$$\Phi_E = E \cdot A \quad \leftarrow \text{not quite right}$$



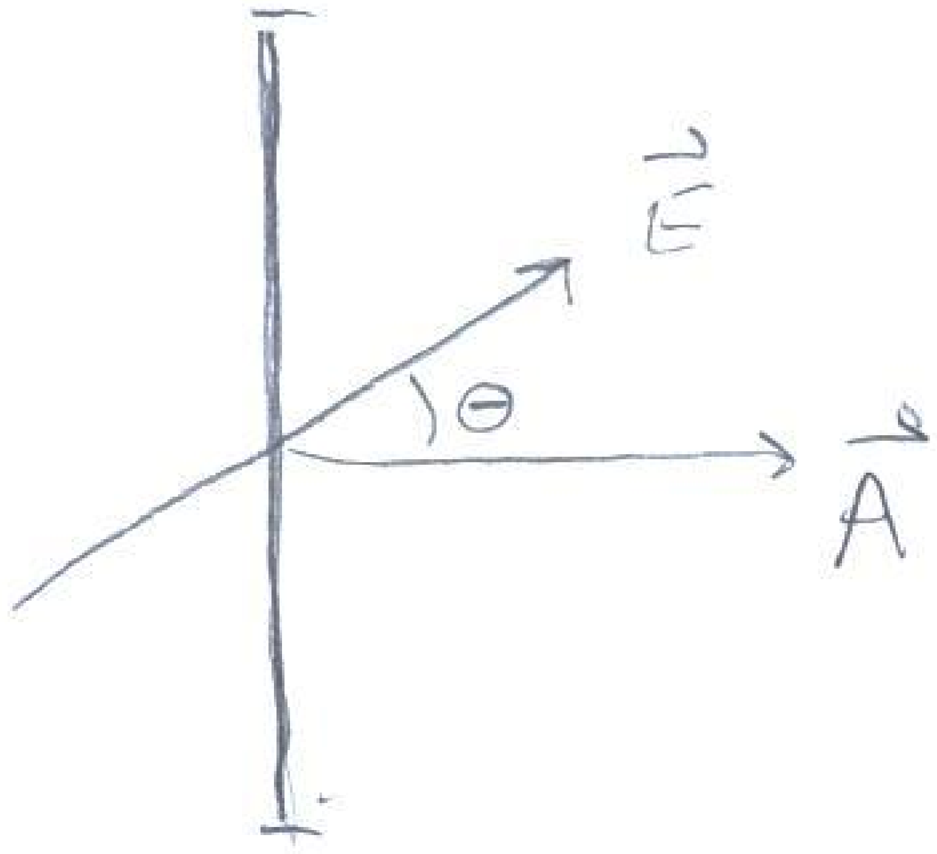
Side View



$$\Phi_E = \left(\frac{\overset{E_{\text{field}}}{\text{number of Lines}}}{\text{per area}}\right) \times \text{Area}$$

$$= \#\text{ of lines crossing a surface}$$

If the electric Field is not perpendicular
To the normal

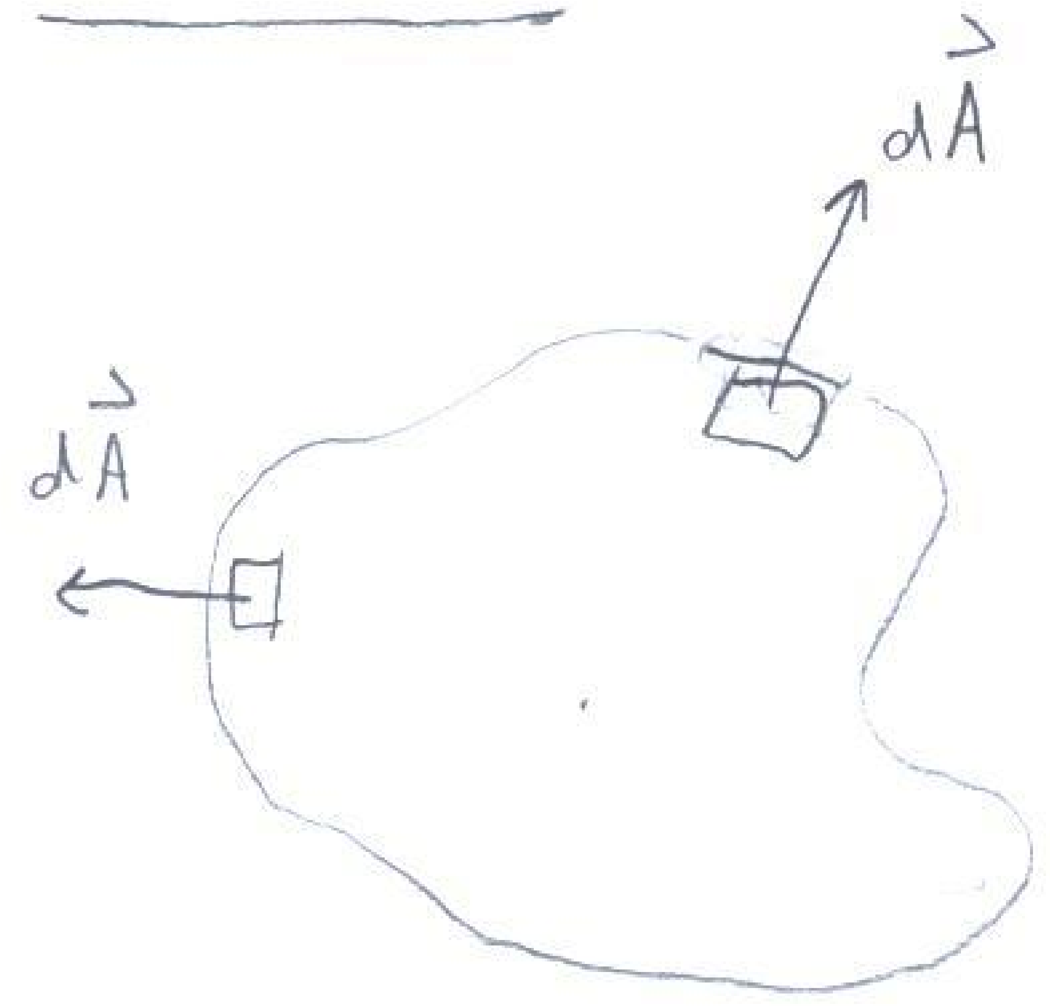


$$\Phi_E \equiv E A \cos\theta$$

$$\Phi_E = \vec{E} \cdot \vec{A} \leftarrow \text{has magnit}$$
of are

vector which is normal to surface

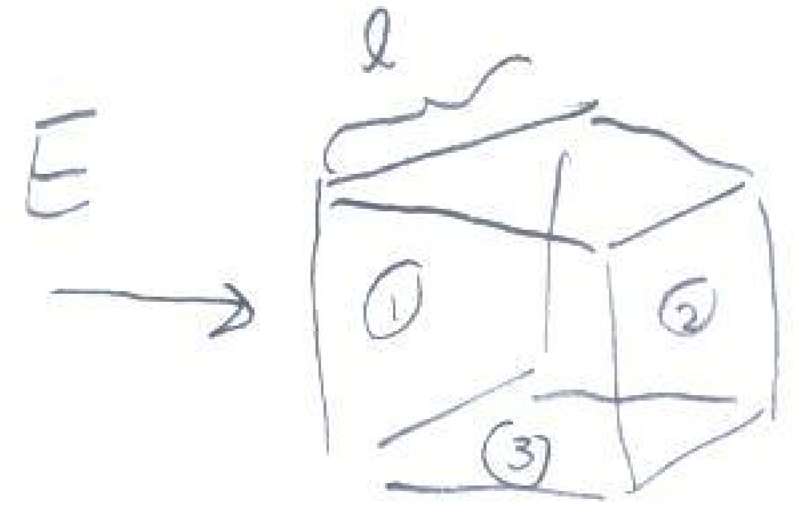
In General



$$\Phi_E = \sum_i \vec{E}\cdot d\vec{A} = \int_{\text{Surface}} \vec{E}\cdot d\vec{A}$$

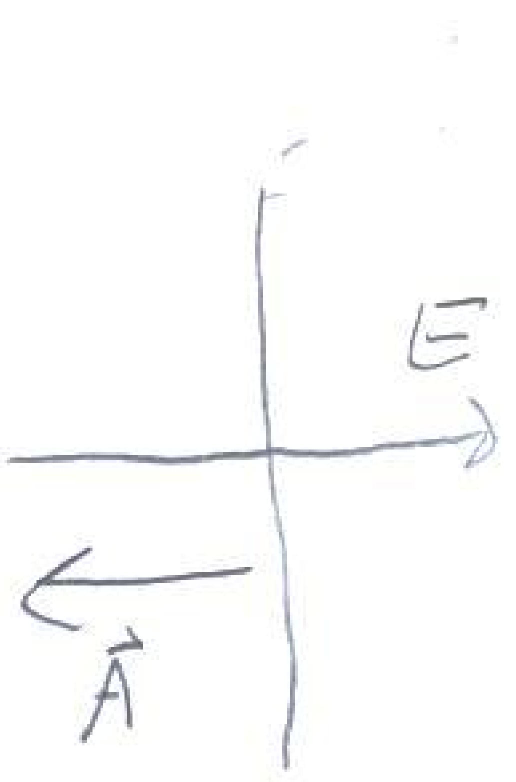
= number of lines leaving a surface - lines entering

Examples:



- A cube of side l

Calculate The Flux through ①



$$\Phi_E \text{ through ①} = \vec{E}\cdot\vec{A}$$

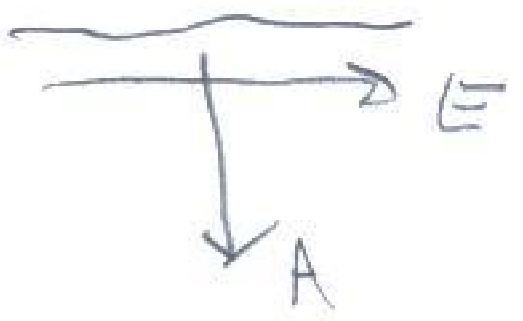
$$= EA\cos 180^\circ = -EA$$

$$= -El^2$$

Then Φ_E through ②

$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos 0^\circ = EA = E\ell^2$$

Then Φ_E through ③



$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos 90^\circ$$

$$= 0$$

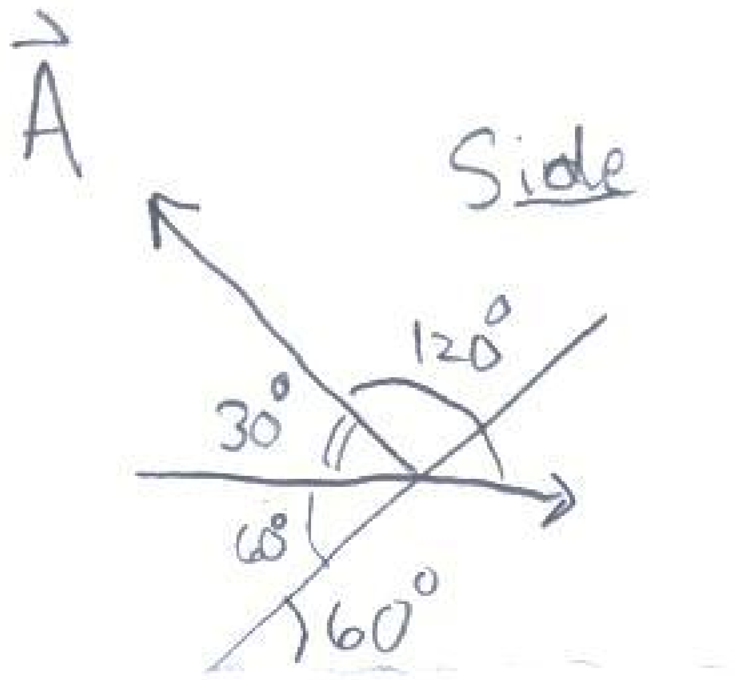
Then Φ_E other faces are zero because $\cos\theta = 0$

$$\Phi_{E,TOT} = \Phi_{E_{\textcircled{1}}} + \Phi_{E_{\textcircled{2}}} + \Phi_{E,\textcircled{3}} + \Phi_{E,\textcircled{4}} + \Phi_{E,\textcircled{5}} + \Phi_{E,\textcircled{6}}$$

$$= -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0$$

$$= 0$$

Consider:

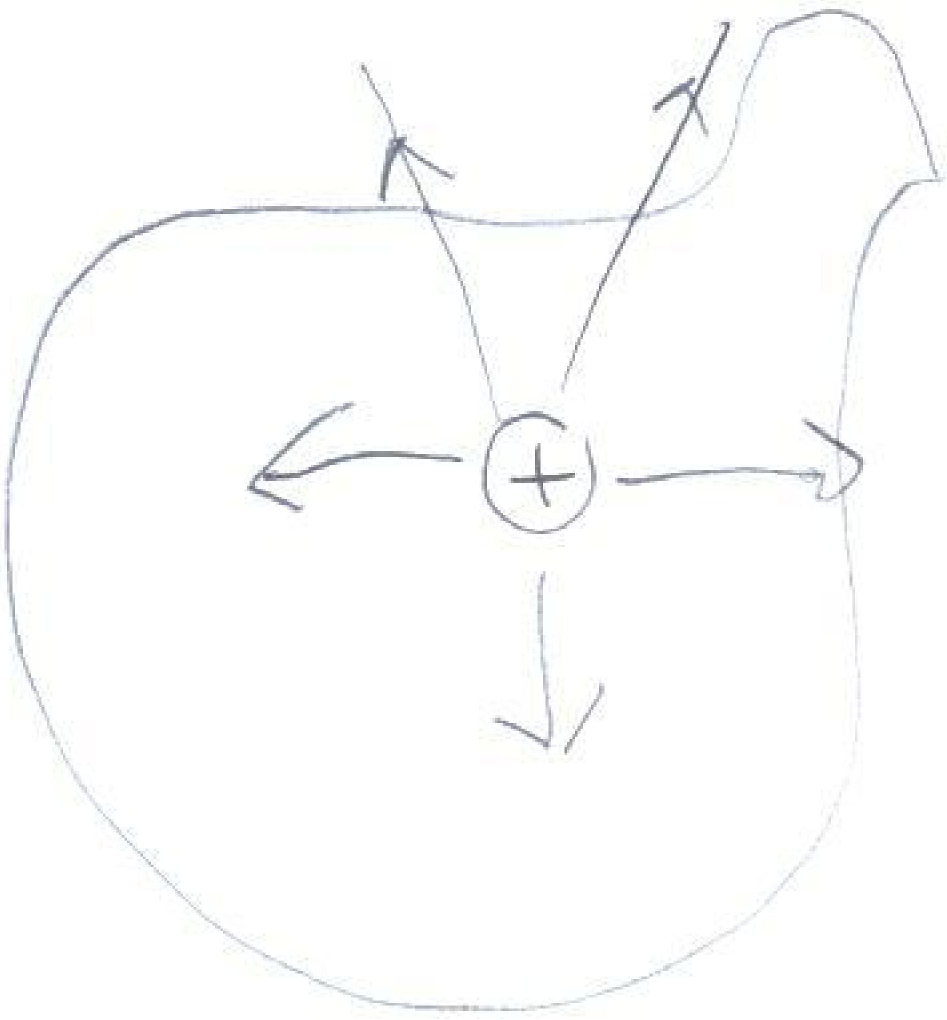


$$\Phi_E = \vec{E} \cdot \vec{A}$$

$$\Phi_E = E\,A \cos 120^\circ = - EA \cos 30^\circ$$

Gauss Law

- Recall Field lines

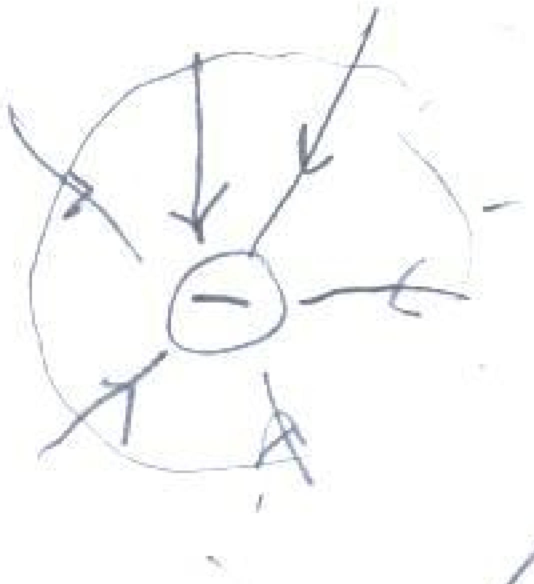


$\Phi_E =$ number of lines which leave the surfa

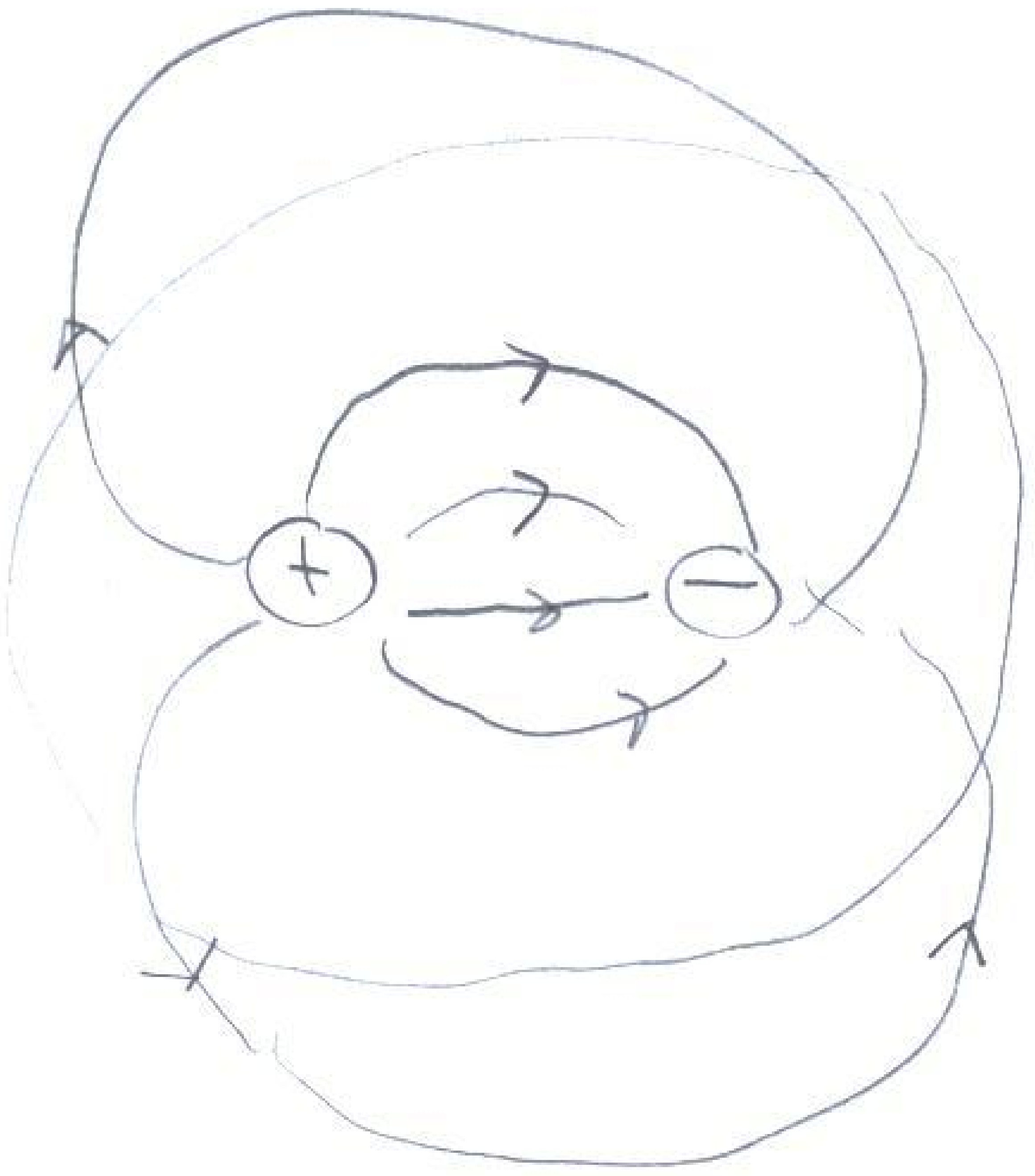
$\propto$ Charge inside

Answer

$$\Phi_E = \int \vec{E} \cdot d\vec{A} = 4\pi k_e \left(\text{net charge inside} \right)$$



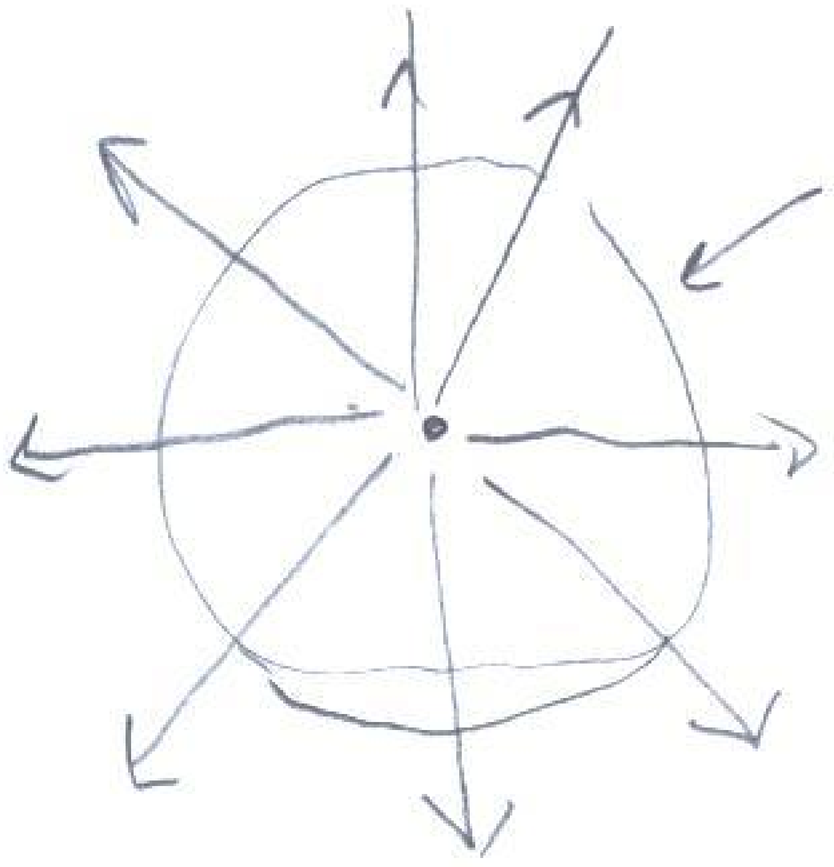
Φ_E = − number of lines entering



$\Phi_E = 0$. { For every Entering Field line there is an exiting li

Using Gauss Law to Find E-field

Ex1 Coulomb



make believe "gaussian surface"

$$\Phi_E = \int \vec{E}\cdot d\vec{A} = EA = E_r 4\pi r^2$$

symmetry

$$\Phi_E = \underbrace{4\pi k_e}_{} q \leftarrow \text{Gauss Law}$$

$$E_r 4\pi r^2 = 4\pi k_e q$$

$$E_r = \frac{k_e q}{r^2}$$

Redo

$$k_e = \frac{1}{4\pi \varepsilon_0}$$

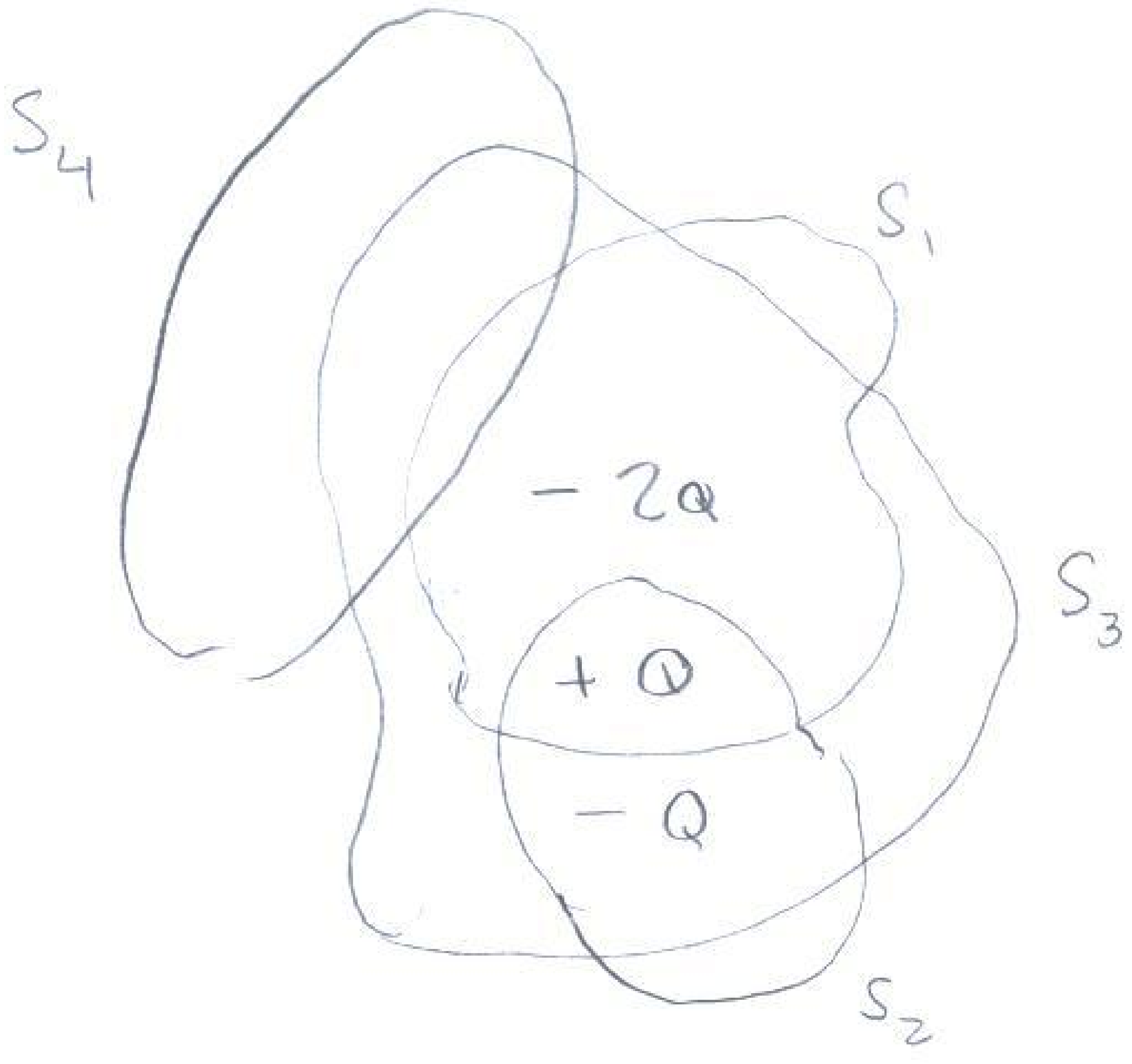
$$\Phi_E = \frac{q}{\varepsilon_0}$$

$$E 4\pi r^2 = \frac{q}{\varepsilon_0} \longrightarrow \epsilon = \frac{1}{4\pi \varepsilon_0}\frac{q}{r^2}$$

Which Could be true or Must be True?

The Flux is zero

	Could be	Must be
$E = 0$	✓	
The charge inside = 0	✓	
The net charge = 0		✓
The number of field lines entering = # lines exiting		✓



Calculate the Flux through the different surfaces

Gauss Law For a sphere

$$\rho = \frac{Q}{V} = \frac{\text{Charge}}{\text{Volume}} = \frac{Q}{\frac{4}{3}\pi a^3}$$

a) Find the Electric Field outside the sphere

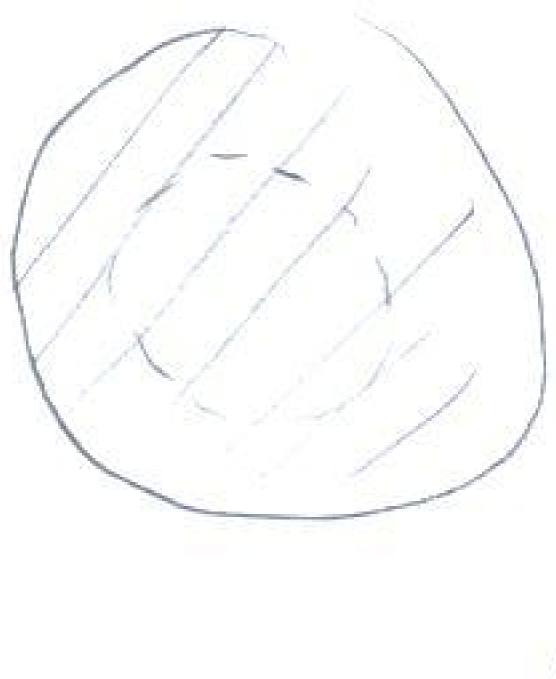
$$\Phi_E = \int E \cdot d\vec{A} = E_r 4\pi r^2$$

$$\Phi_E = 4\pi k_e (q\,\text{net})$$

$$E_r \cdot 4\pi r^2 = 4\pi k_e (+Q)$$

$$E_r = k_e \frac{Q}{r^2} \leftarrow \text{Outside}$$

b)



$$\Phi_E = \int \vec{E} \cdot d\vec{A} = E_r \cdot 4\pi r^2$$

$$\Phi_E = 4\pi k_e (q\,\text{net inside})$$

$$E_r 4\pi r^2 = 4\pi k_e \left(\rho \frac{4}{3}\pi r^3\right)$$

So

$$E_r = k_e \rho \frac{4\pi}{3} r$$

$$E_r = k_e \frac{Q}{\frac{4}{3}\pi a^3} \frac{4\pi}{3} r$$

$$E_r = \frac{k_e Q}{a^2} \cdot \left(\frac{r}{a}\right)$$ ← inside

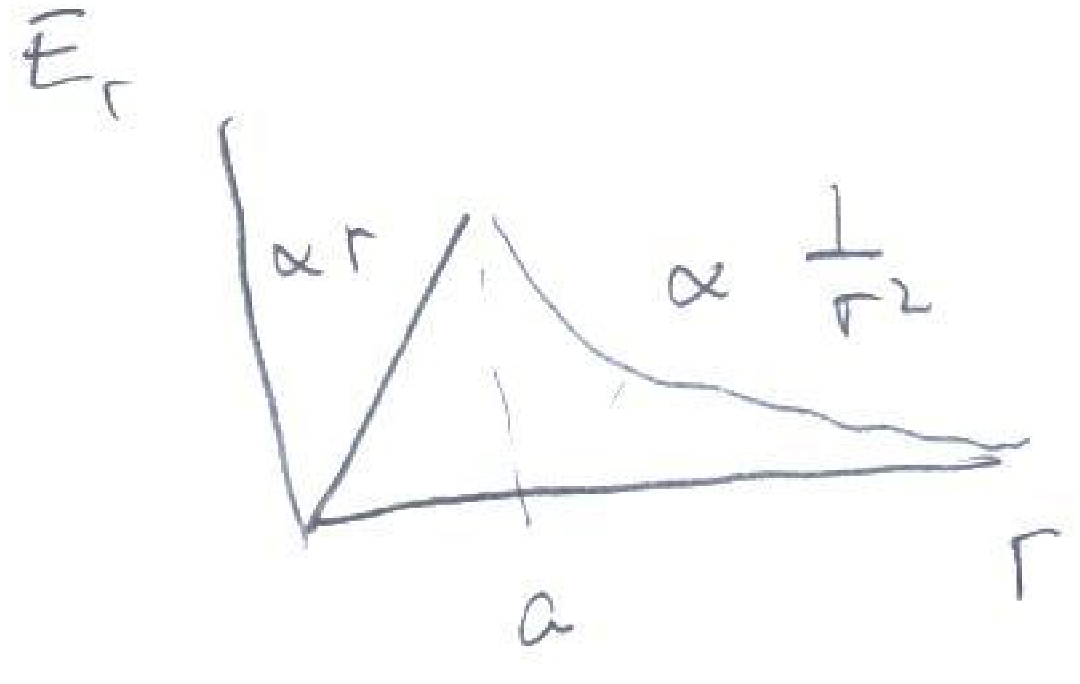
What about when neither inside or out,
i.e $r = a$

$$E_r = \frac{k_e Q}{a^2}$$ ← outside formula

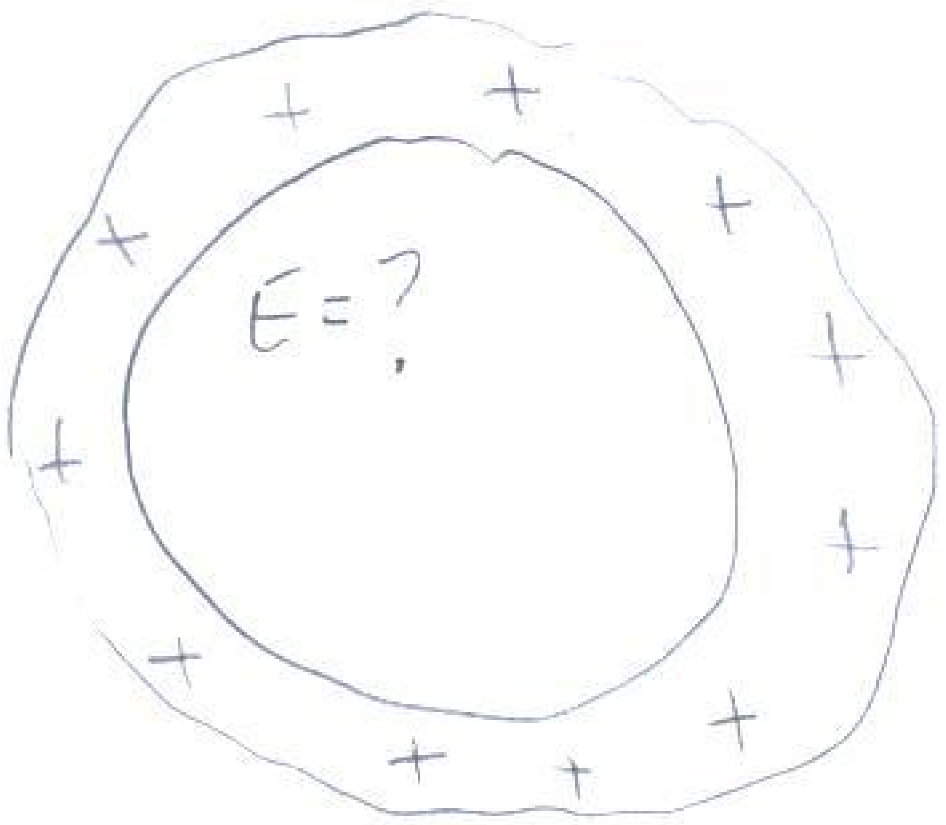
$$E_r = \frac{k_e Q}{a^2} \cdot \left(\frac{a}{a}\right)$$ ← inside formula

these agree

Graph



Then Consider



① Draw a surface

② Calculate the flux

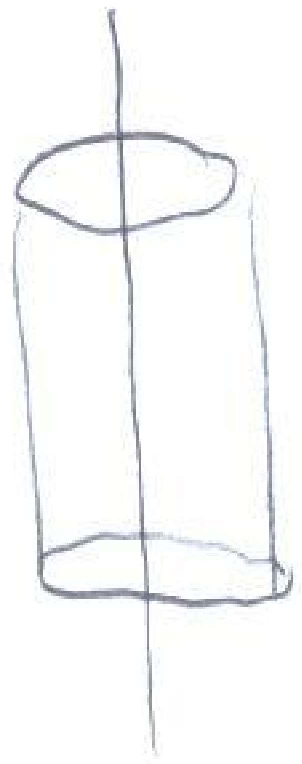
$$\Phi_E = E_r 4\pi r^2 = 0$$

$$E_r = 0$$ inside,

$$\Phi_E^{\text{outside}} = E_r 4\pi r^2 = 4\pi k_e Q$$

$$E_r = \frac{k_e Q}{r^2}$$

Line Of Charge:



$$\Phi_E = E_\perp \cdot A$$

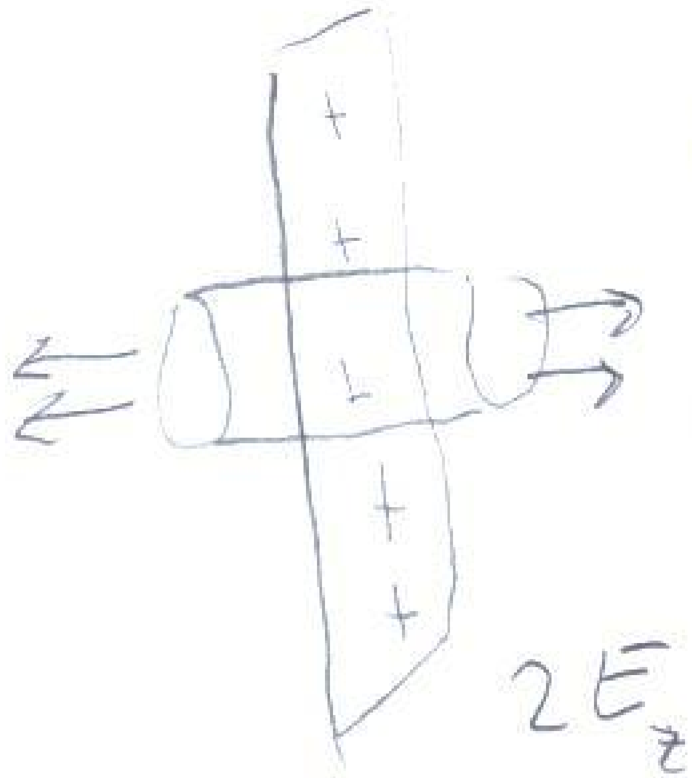
$$\Phi_E = E_r 2\pi r \cdot L$$

$$\Phi_E = 4\pi k_e Q$$

$$E_r \cdot 2\pi r L = 4\pi \cdot k_e \lambda \cdot L$$

$$\boxed{E_r = \frac{2k_e\lambda}{r}}$$

Infinite Wall of Charge



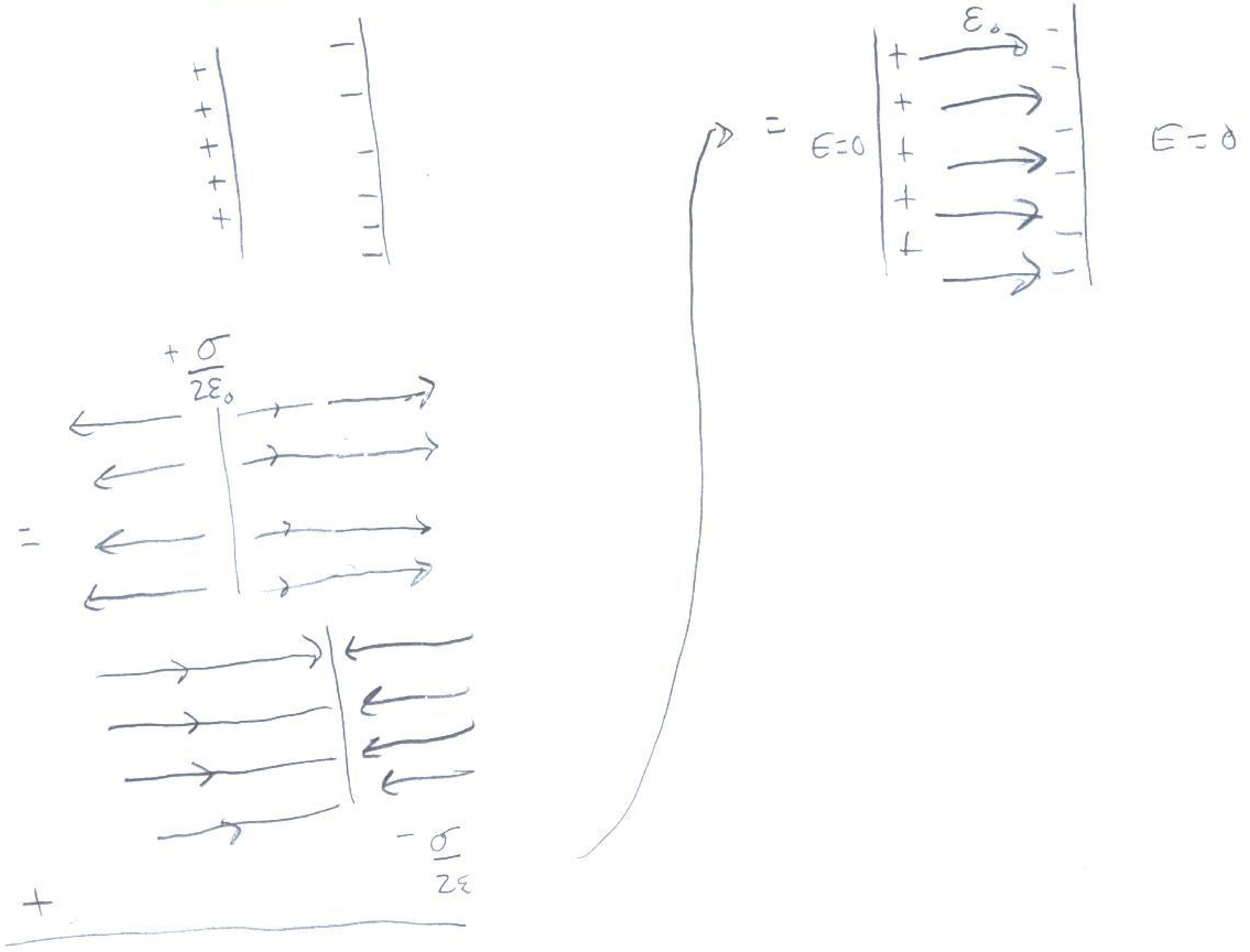
$$\Phi_E = 2E_z \cdot A$$

$$\Phi_E = 4\pi k_e \sigma \cdot A$$

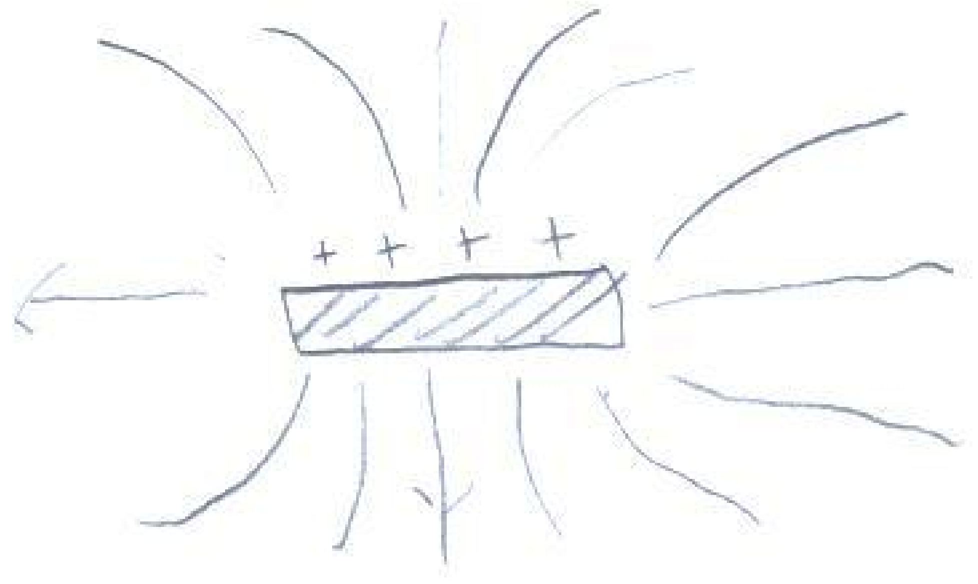
$$2E_z A = 4\pi k_e \sigma A$$

$$E_z = 2\pi k_e \sigma = \frac{\sigma}{2\varepsilon_0}$$

Suppose Then



Consider A finite line of Charge:

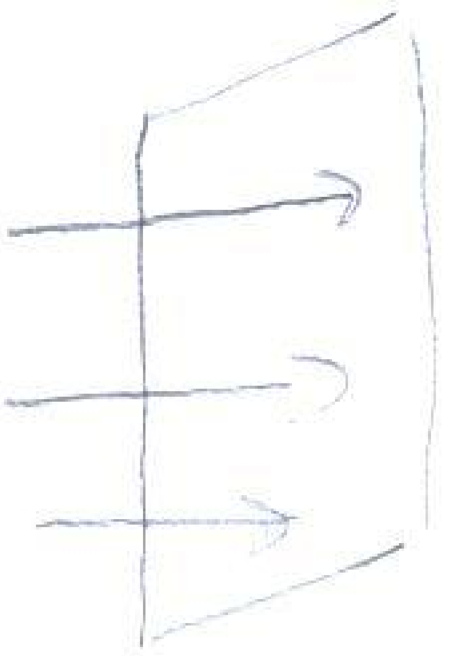


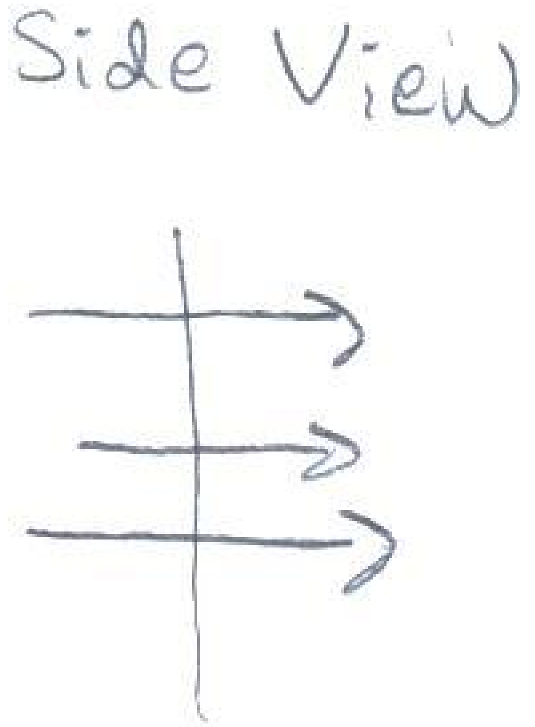
Gauss's Law Can be used, but you cant the Electric Field From it

$$\int \vec{E} \cdot d\vec{A} \neq E(A)$$

Electric Flux

$\Phi_E = E \cdot A$ ← not quite right





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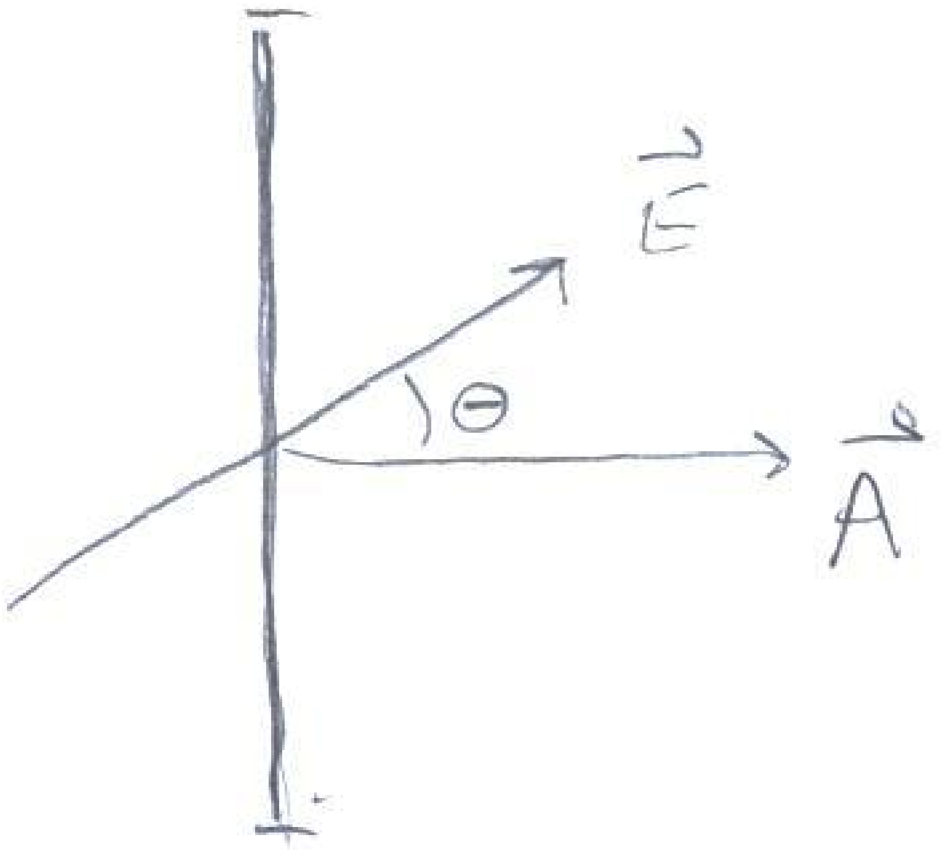
= # of lines crossing a surface

If the electric Field is not perpendicular To the normal



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vector which is normal to surface