Last Time, Waves:

- $y(x,t) = f(x \mp vt)$

- $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{V^2}\dfrac{\partial^2 y}{\partial t^2}$

Wave equation gives th

Harmonic Waves:

- $A \sin(kx \mp \omega t + \phi)$

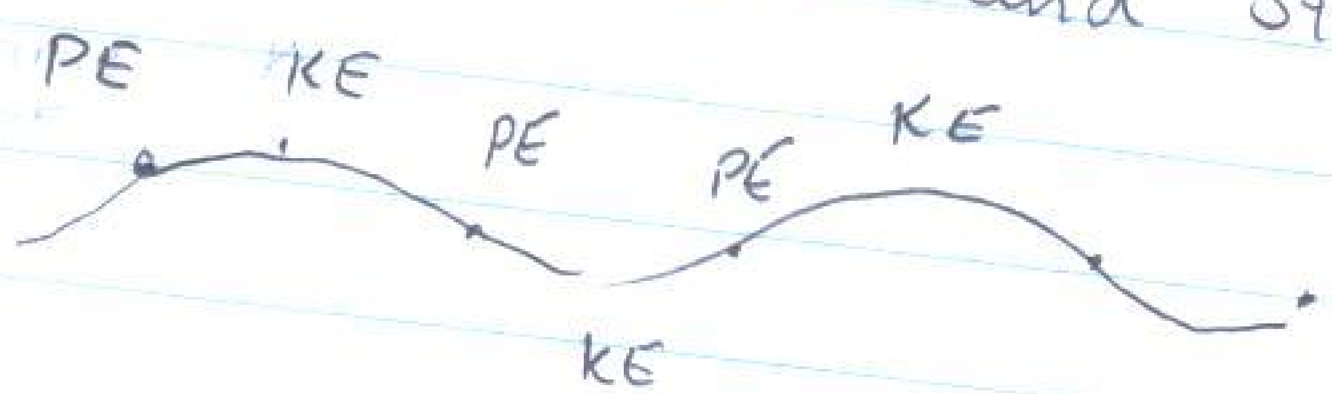
$k = \dfrac{2\pi}{\lambda}$ — wavelength

$\omega = \dfrac{2\pi}{\tau}$ — perio

Speed: $V = \lambda f$

$V = \dfrac{\omega}{k}$

Newtons Laws and String



$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} \qquad v^2 = \frac{T}{\mu}$$

μ = mass per length

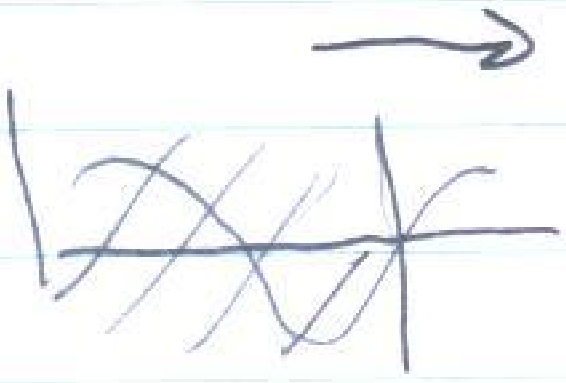
T = Tension in rope

$\dfrac{dPE}{dx}$ = Stretching $= \dfrac{1}{2}T\left(\dfrac{\partial y}{\partial x}\right)^2 \xrightarrow[\text{harmonic}]{} \left\langle \dfrac{dPE}{dx} \right\rangle = \dfrac{1}{4}\mu\omega^2 A$

$\dfrac{dKE}{dx}$ = up down $= \dfrac{1}{2}\mu\left(\dfrac{\partial y}{\partial t}\right)^2 \xrightarrow[\text{harmonic}]{} \left\langle \dfrac{dKE}{dx} \right\rangle = \dfrac{1}{4}\mu\omega^2 A^2$

P.

Power:

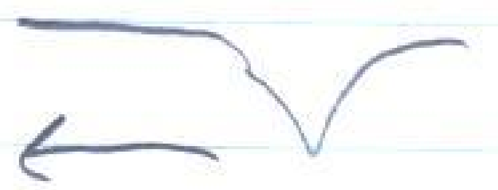


$$\frac{dx}{dt}\left\langle \frac{dKE}{dx} + \frac{dPE}{dx} \right\rangle = \pm\frac{1}{2}\mu\omega^2 A^2 V$$

Today

- Retlection of waves
- Sound Waves
  - → qaulitative
  - → spkerical waves
  - → dB
- Superposition
  - → general sin
  - → Standing wave
    - → Energetics
    - → power transfer
  - → waves on a string
  - → waves in a piple

Reflection, Fixed End

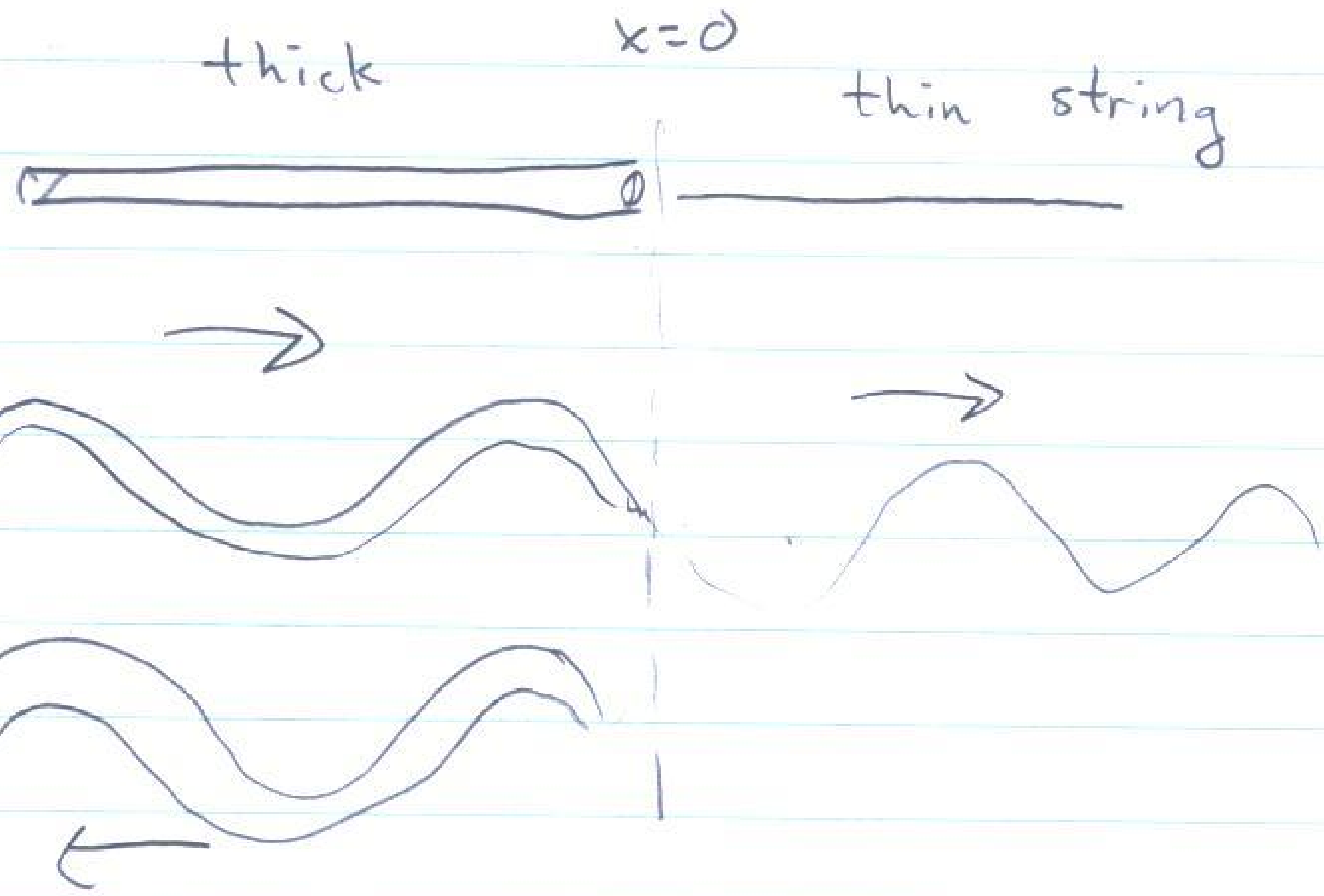


amplituder Amplitude inverted

Reflection Free end;

not inverted

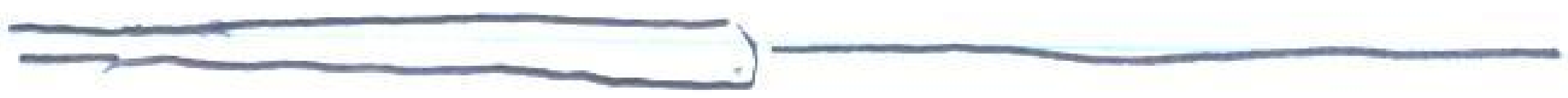
Then Consider The general problem



The original wave is partly reflected and partly transmitted

Reflection

μ_L, c_L $\qquad\qquad$ μ_R, c_R



$$y_L = A \sin(k_L x - \omega t) + \underbrace{A_r \sin(-k_L x - \omega t)}$$

reflected wave
with same time
dependenc

$$y_R = A_T \sin(k_R x - \omega t)$$

$$y_L\Big|_{x=0} = y_R\Big|_{x=0}$$

(1) $A + A_R = A_T$ $\qquad$ gives

$$\frac{\partial y_L}{\partial x}\Big|_{x=0} = \frac{\partial y_R}{\partial x}\Big|_{x=0} \qquad \text{gives}$$

(2) $A k_L - A_r k_L = A_T k_R$

$$(A - A_r)\cancel{k_L} = A_T \frac{c_L}{c_R} \cancel{k_L}$$

$$k_R = \frac{\omega}{V_R} = \frac{V_L}{V_R} \underbrace{\frac{\omega}{V_L}}_{k_L}$$

$$k_R = k_L \frac{c_L}{c_R}$$

From

(1) & (2) we get

$$\frac{A_r}{A} = \frac{c_R - c_L}{c_R + c_L} = \frac{\sqrt{\mu_L} - \sqrt{\mu_R}}{\sqrt{\mu_R} + \sqrt{\mu_L}}$$

$$\frac{A_T}{A} = \frac{2}{1 + c_L/c_R} = \frac{2}{1 + \sqrt{\mu_R/\mu_L}}$$

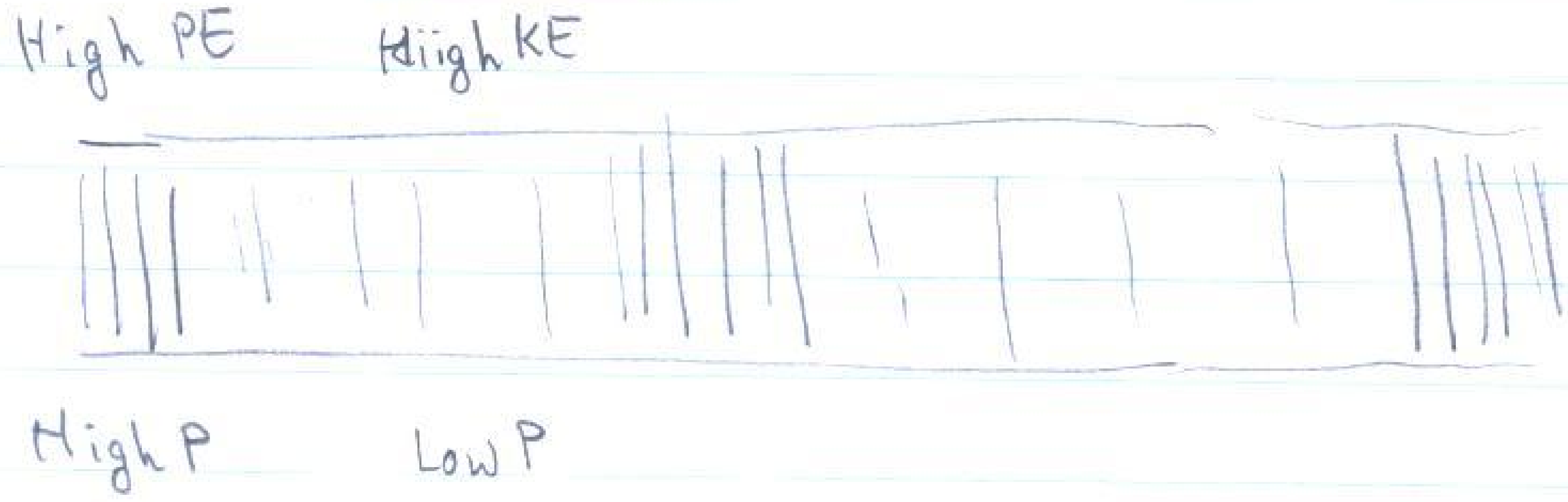
$$2A = A_T\,(1 + c_L/c_R) \Rightarrow A_T = \frac{2}{1+c_L/c_R}\,A$$

$$A + A_R = A_T$$

$$A_R = \left(\frac{2}{1+c_L/c_R} - 1\right) A$$

$$A_R = \left(\frac{1 - c_L/c_R}{1 + c_L/c_R}\right) A$$

Sound waves



(1) A Longitudinal wave, of pressure or density

density

Sound speed ~340 m/s)

$$\frac{\partial^2 \rho(x,t)}{\partial t^2} = V_s^2 \frac{\partial^2 \rho}{\partial x^2}$$

$$V_s = \sqrt{\frac{B}{\rho}}$$

analog of T

analog of μ

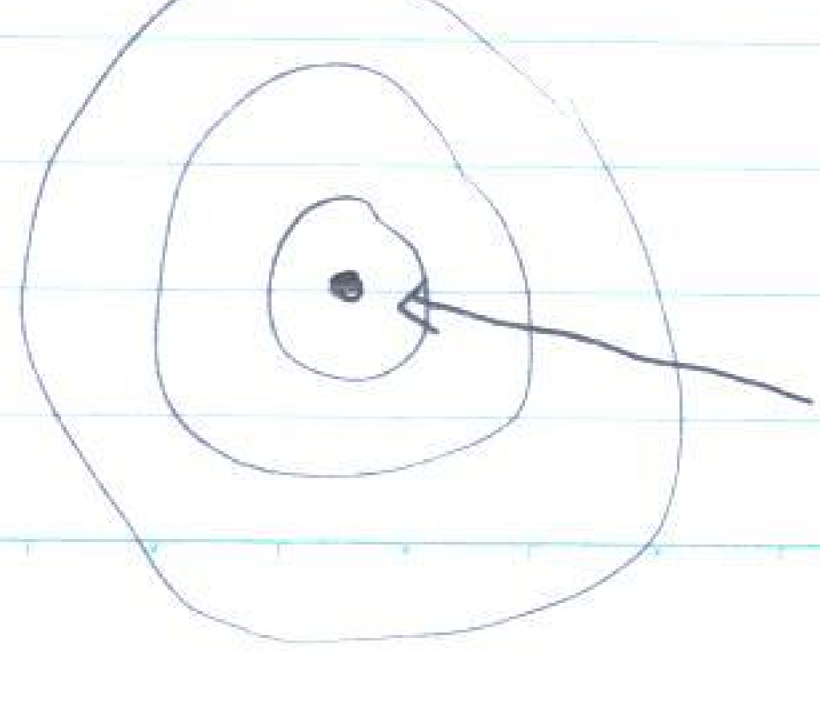
Bulk modulus

$$P = -B \frac{\Delta V}{V}$$

- $C_s \sim 340 m/s$ $\omega \sim 440 Hz$

(2) Spherical Waves

$$P \propto \frac{C \sin(kr - \omega t)}{r}$$



total power spreads out evenly

$$\text{Intensity} = \frac{\text{Power}}{\text{Area}} \propto (\text{pressure})^2 = \frac{(\Delta P_{max})^2}{2\rho_0 v_s}$$

$$I = \frac{\mathcal{P}}{4\pi r^2}$$

$$\beta = \text{Sound level} = 10 \log_{10} \frac{I}{I_0}$$

$I_0 = 1 \times 10^{-12}\ W/m^2$

Example

- A point source emits with $\mathcal{P} = 80.0\ W$ find the intensity 3m away

$$I = \frac{\mathcal{P}}{4\pi r^2} = \frac{80.0\ W}{4\pi (3.0\ m)^2} = 0.707\ W/m^2$$

What is the sound level

- Find the distance where I is $1 \times 10^{-8}\ W/m^2$. What the is the sound level ?

$$I = \frac{\mathcal{P}}{4\pi r^2} \Rightarrow r = \sqrt{\frac{\mathcal{P}}{4\pi I}} = \sqrt{\frac{80.0\,W}{4\pi(1.\times 10^{-8}\ W/m^2)}}$$

$$r = 2.52 \times 10^4\ m \qquad \sim 16\ mi$$

1 mile 5280 ft

- For, $I = 0.707\ W/m^2$, we have

$$\beta = 10 \log_{10}\left(\frac{0.707\ W/m^2}{1\times 10^{-12}\ W/m^2}\right)$$

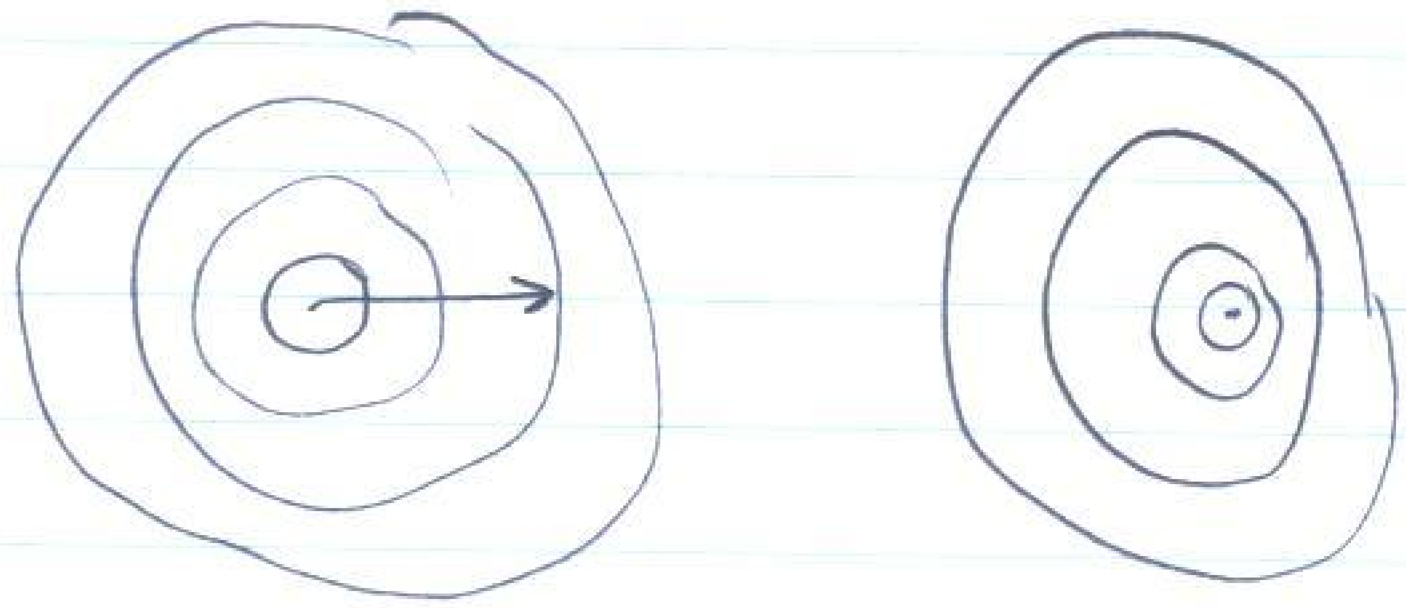
$$\left.\begin{aligned} \beta &= 10\left[\log_{10}(0.7) + 12\right] \\ &= 10\left[-0.15 + 12\right] \end{aligned}\right\} \text{Rock Concert}$$

$$\beta = 118\ dB$$

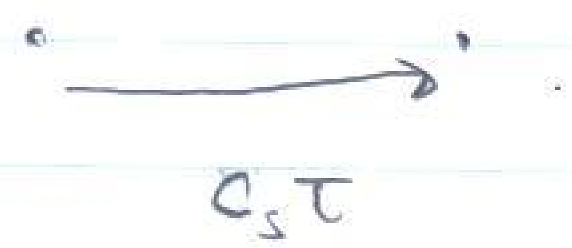
- For $I = 1\times 10^{-8}\ W/m^2$

$$\beta = 10 \log_{10}\left(\frac{1\times 10^{-8}\ W/m^2}{1\times 10^{-12}\ W/m^2}\right) = \underbrace{10 \cdot 4 = 40\ dB}_{\text{mosquito buzzing}}$$

Doppler Effect



At Rest



Moving

$\lambda' = (c_s - v_s)\tau$, so , $f' = \dfrac{c_s}{\lambda} = \dfrac{c_s}{(c_s - V_s)\tau}$

$$\boxed{f' = \frac{c_s}{c_s - v_s} f}$$ source moving toward observer

$f' = \dfrac{c_s}{(c_s + V_s)} f$ source moving away from observer

Time between observations $= \tau' = \dfrac{c_s \tau}{c_s + v_o}$

$$f' = \left(\frac{c_s + v_o}{c_s}\right) f$$

$$f' = \left(\frac{c_s + v_o}{c_s} \right) f$$ frequency oberved by an observer approaching the fource

Finally summary:

$$f' = \left(\frac{c_s + v_o}{c_s - v_{sr}} \right) f$$

Example Problems

P.39 Ch 17

You

ambulance ← 480 Hz

ambulace 560 Hz ←

Determine the ambulance speed

Solution

$$f'_A = \left(\frac{c_s}{c_s - v_s} \right) f_o$$

$$f'_B = \left(\frac{c_s}{c_s + v_s} \right) f_o$$

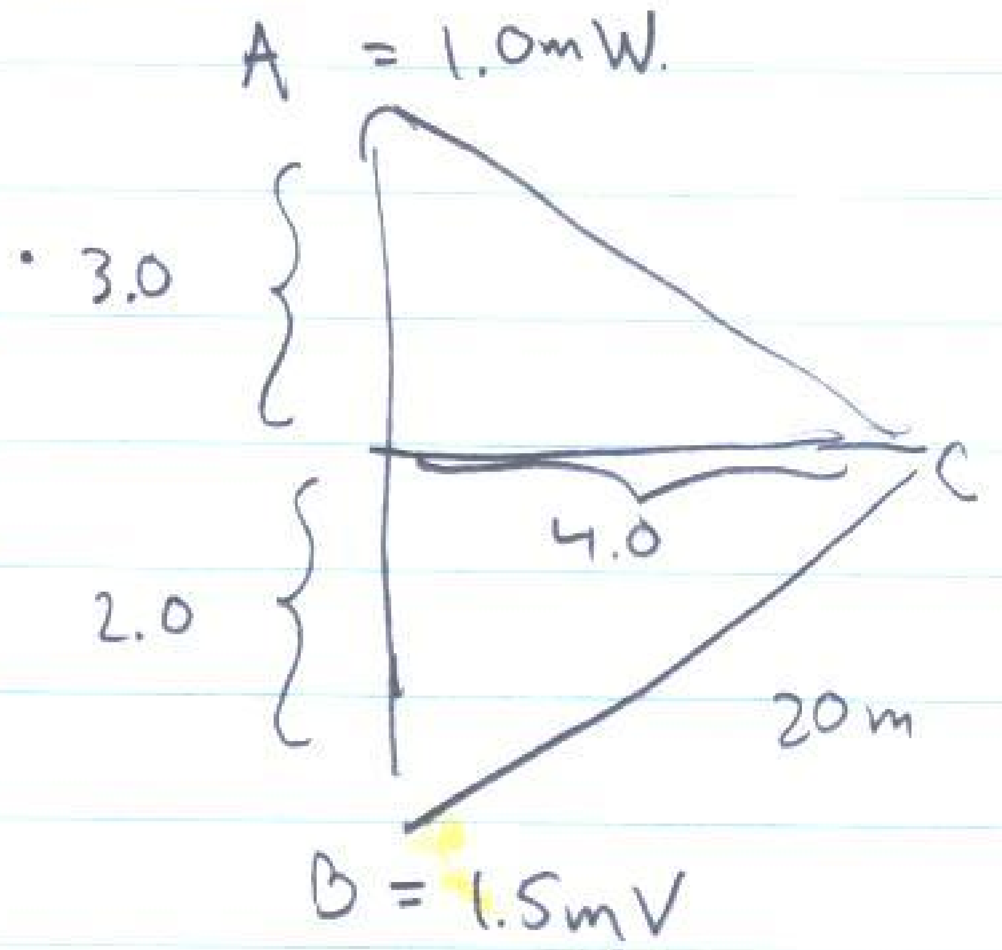
$$\frac{f'_A}{f'_B} = \frac{c_s + v_s}{c_s - v_s} = \frac{(1 + v_s/c_s)}{(1 - v_s/c_s)}$$

$$\left(1 - \frac{v_s}{c_s}\right)(f'_A/f'_B) = (1 + v_s/c_s)$$

$$\frac{f_A'/f_B' - 1}{(f_A'/f_B' + 1)} = v_s/c_s$$

$$26.4\ \text{m/s} = v_s$$

<u>Problem 31</u> <u>Chp 17</u>



- A only = 65 dB
- B only = 67.8 dB
- A + B = 69.6 dB

Superposition of Waves

$$\frac{\partial^2 y_1}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y_1}{\partial t^2}$$

$$\frac{\partial^2}{\partial x^2}(y_1 + y_2) = \frac{1}{V^2}\frac{\partial^2}{\partial t^2}(y_1 + y_2)$$

- If y_1 and y_2 are solutions to the wave eqn then so is $y_1 + y_2$

$$y = A\sin(kx - \omega t) + A\sin(kx - \omega t + \phi)$$

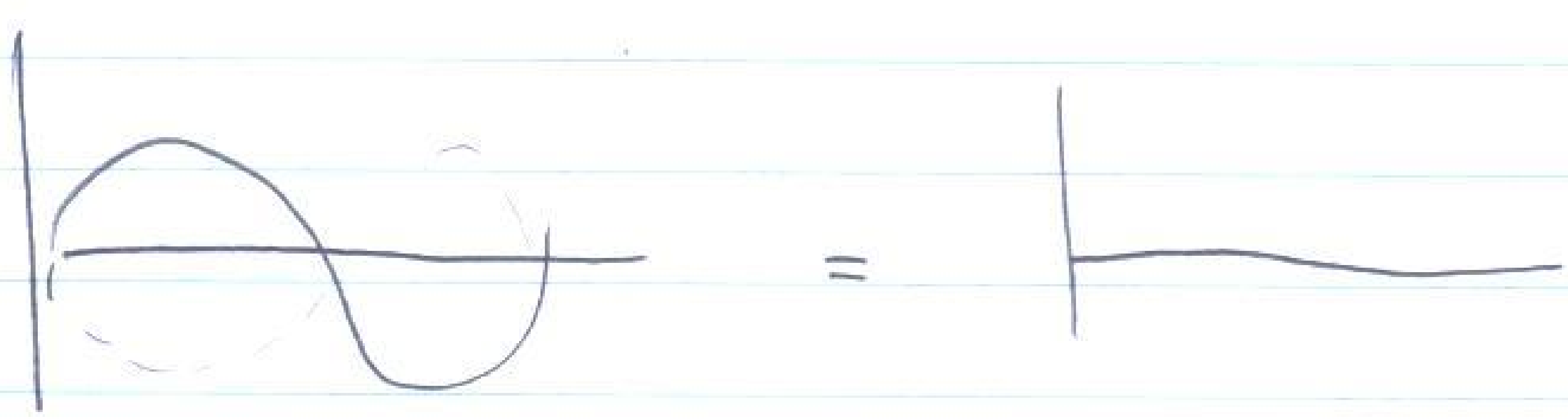
$$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$$

$$\sin(a+b) + \sin(a-b) = 2\sin a \cos b$$

$$\sin a' + \sin(b') = 2\sin\left(\frac{a'+b'}{2}\right)\cos\left(\frac{a'-b'}{2}\right)$$

$$a' = kx - \omega t \qquad b' = kx - \omega t + \phi$$

$$y_1 + y_2 = 2A\cos\phi \sin(kx - \omega t + \phi)$$



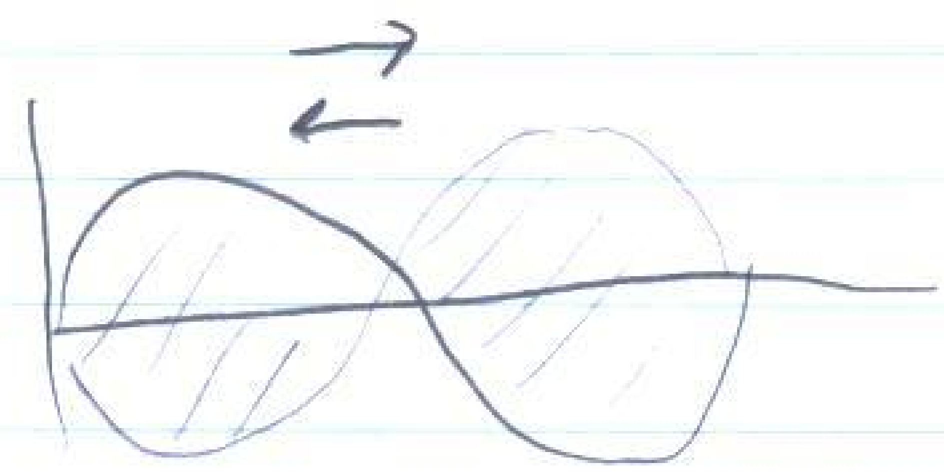
Standing Waves

$$y = f(x-vt) + f(x+vt)$$

$$y = A\sin(kx-\omega t) + A\sin(kx+\omega t)$$

$$\sin(a+b) + \sin(a-b) = 2\sin a\cos b$$

$$y = 2A\sin kx\cos\omega t$$



- Energetics:

$$P = \frac{1}{2}\mu\omega^2 A v - \frac{1}{2}\mu\omega^2 A v = 0$$

Last Time, Waves:

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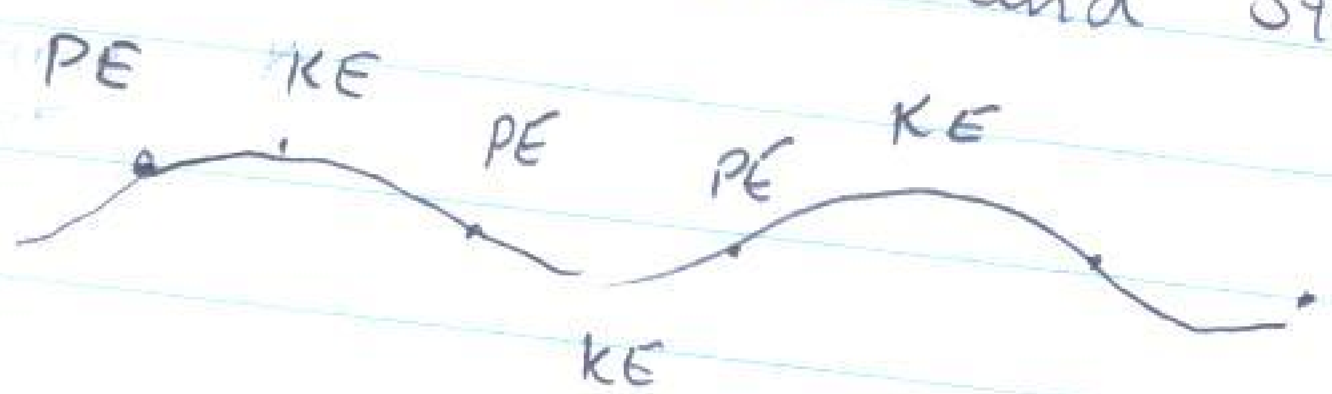
Speed: $v = \lambda f$

$v = \dfrac{\omega}{k}$

Newtons Laws and String



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$\dfrac{dKE}{dx}$ = up down $= \dfrac{1}{2} \mu \left(\dfrac{\partial y}{\partial t}\right)^2 \xrightarrow{\text{harmonic}} \left\langle \dfrac{dKE}{dx} \right\rangle = \dfrac{1}{4} \mu \omega^2 A^2$

P.