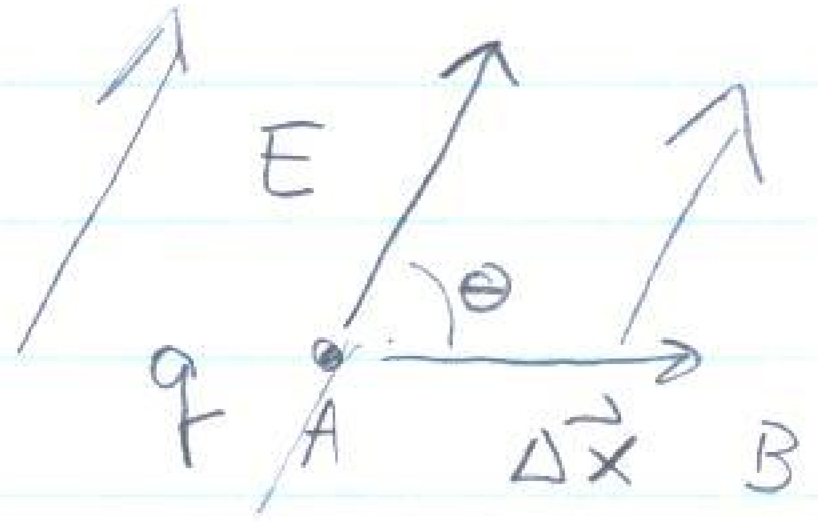
Electrostatic Potential

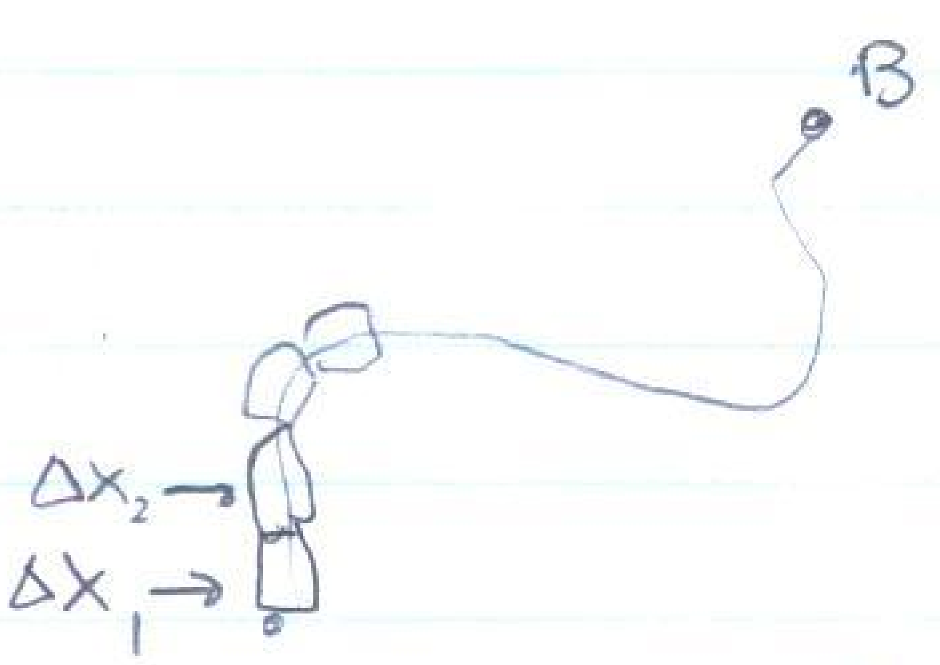


ΔW_{AB} = Work done By the Electric Field in moving charge from A to B

$$= \vec{F} \cdot \Delta\vec{x} = |\vec{F}||\Delta x| \cos\theta$$

$$\Delta W_{AB} = q \vec{E} \cdot \Delta \vec{x}$$

More general path



$$W_{AB} = \sum \Delta W_{AB}$$

$$= \sum_{\Delta x} q \vec{E} \cdot \Delta\vec{x}$$

$$W_{AB} = \int_A^B q \vec{E} \cdot \vec{\Delta x}$$ ← Independent of Path!

Then:

ΔU = change in PE from A to B

$$\Delta U = -W_{AB}$$

$$\Delta U = -\underbrace{q}_{\text{property of charge}}\underbrace{\int_A^B \vec{E}\cdot\Delta x}_{\text{property of the E-field}}$$

$$\Delta U = q\,\Delta V \Rightarrow \underbrace{\Delta V_{AB} = -\int_A^B \vec{E}\cdot d\vec{x} = V_B - V_A}$$

ΔV = is the potential difference between points A · B

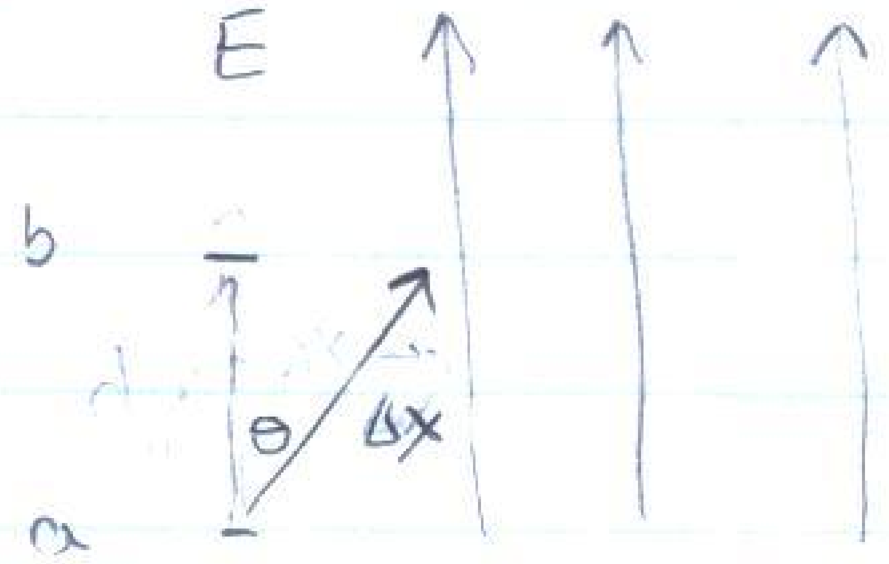
= Also known as the voltage difference

$$\Delta V = \frac{1\,J}{C} = 1 \text{ Volt}$$

Typical Voltages:

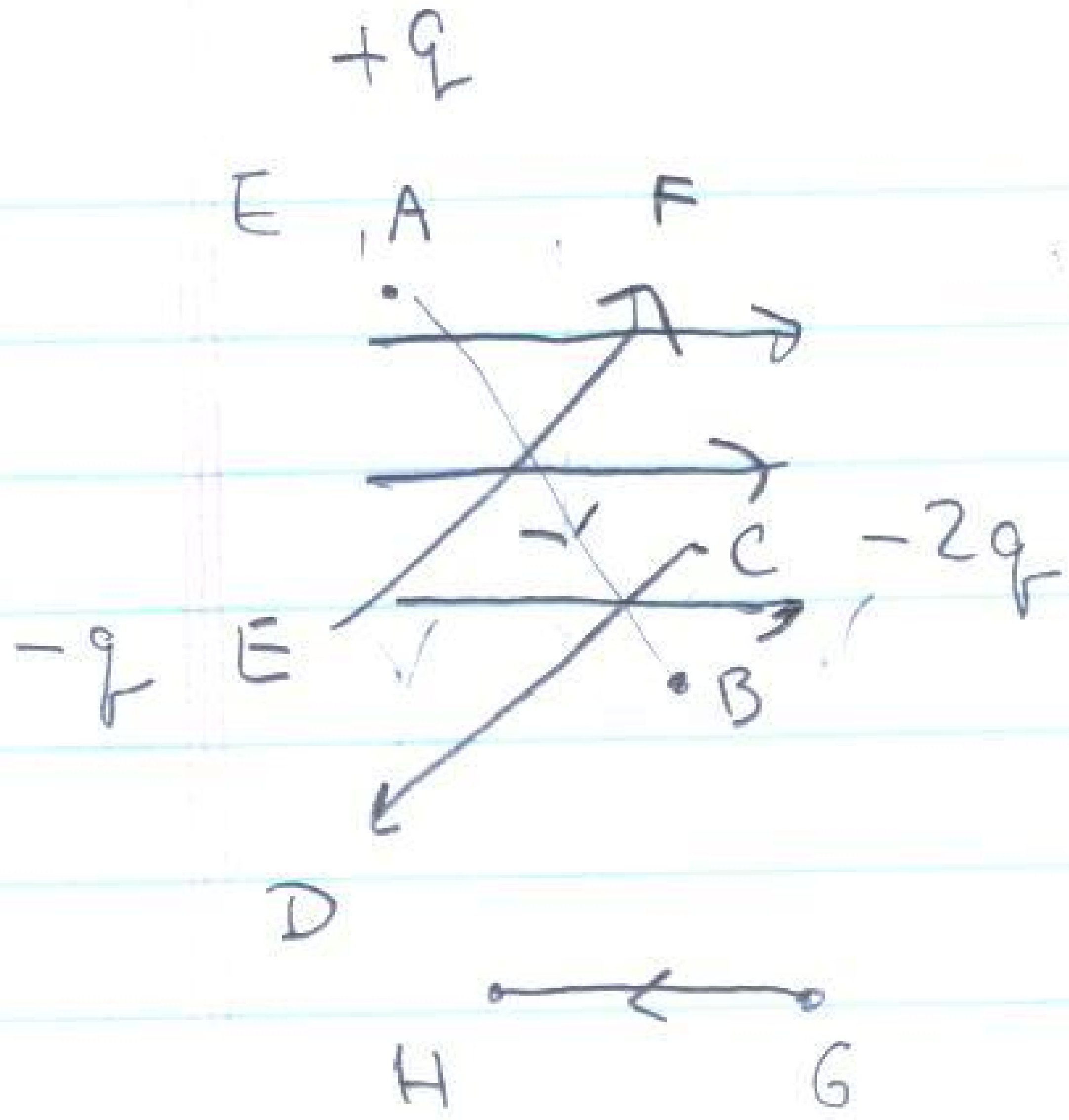
~ 3mV	voltage across membrane of cell
~ 9V	battery
~ kV	Large potentially fatal Voltage ~ old fashioned TV
~ 1000 kV	Huge voltage ↔ largest man-made, Van de Graff

Potential From a uniform field:



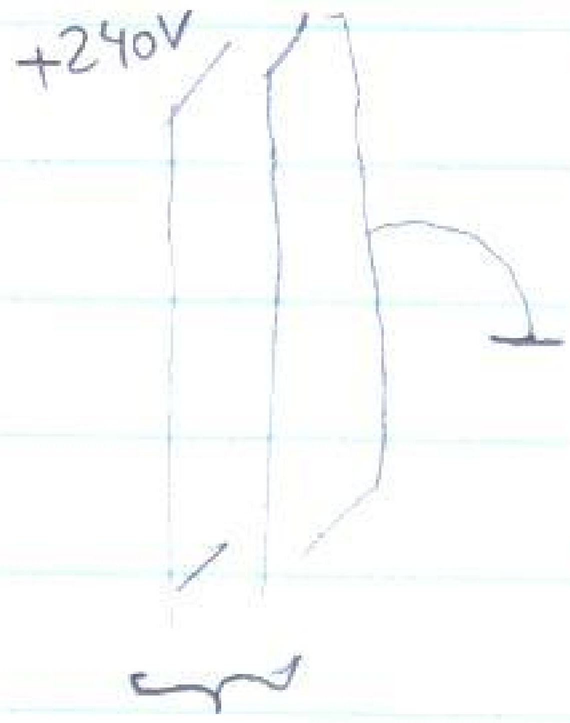
$$\Delta V = -\int_a^b \vec{E} \cdot d\vec{x} = -\vec{E} \cdot \Delta\vec{x} = -\vec{E} \circ \Delta\vec{X}$$

$$\Delta V = -Ed\cos\theta$$



	ΔV ← change in Voltage	ΔU change in
From AB	negative	negative
From CD	positive	negative
From EF	negative	positive
From GH	positive	positive

Typical Electric Fields



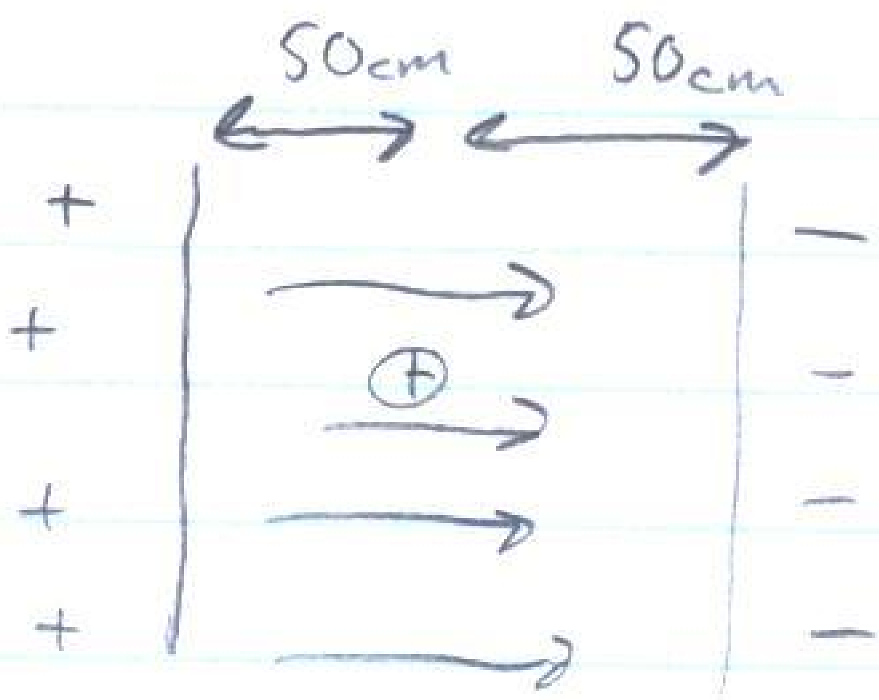
$$|E| = \frac{|V_B - V_A|}{d} = \frac{240V}{0.3 \times 10^{-2}\,m} = 8 \times 10^4 \frac{V}{m}$$

240V

~ sparks at about ~$10^6 \frac{V}{m}$

$d = 0.3$ cm = 3mm

An proton is placed into this field for 50 cm



- Which way does it move
- What is the charge in Potential
- What is the change in Potential Energy
- Find the speed of the proton at the end of its trajectory

• (A)

$$\Delta V = -Ed = -\left(8\times10^{4}\,\frac{V}{m}\right)(0.5\,m) = -4\times10^{4}\,\frac{V}{m}$$

• (B)

$$\Delta U = q\,(\Delta V) = (1.6\times10^{-19}\,C)(-4\times10^{4}\,V)$$

$$= -6.4\times10^{-15}\,J \quad = -4\times10^{4}\,eV$$

• Notice "always" have $1.6\times10^{-19}\,CV \equiv 1eV$

$$\boxed{1.6\times10^{-19}\,J \equiv 1\,eV}$$

• (C) The to find the speed

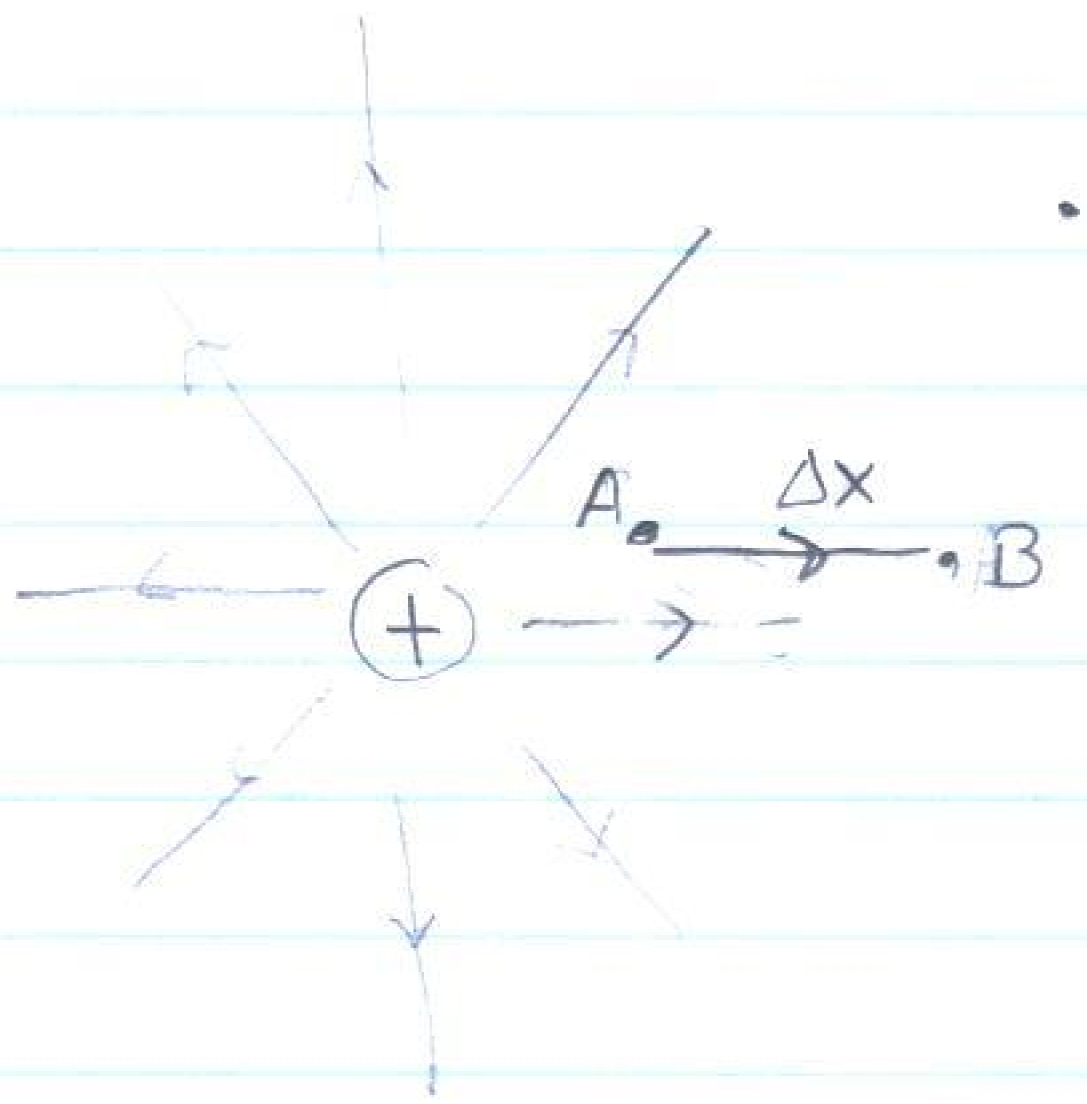
$$0 = \Delta KE + \Delta U \Longrightarrow -(KE_f - \cancel{KE_i}) = \Delta U$$

$$KE_f = -\Delta U$$

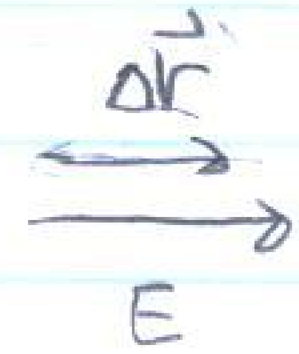
$$KE_f = -(-4\times10^{4}\,eV)$$

$$= 4\times10^{4}\,eV$$

Potential Due to a point charge



- Potential increases from A to B



$$V_{AB} = -\int_A^B \vec{E}\cdot d\vec{x} = -\int_{r_A}^{r_B} \frac{k_e Q}{r^2} \overbrace{\cos 0^\circ}^{(+)} \, dr$$

$$= -\int_{r_A}^{r_B} \frac{k_e Q}{r^2} dr = + \frac{k_e Q}{r} \Bigg|_{r_A}^{r_B}$$

$$V_B - V_A = \frac{k_e Q}{r_B} - \frac{k_e Q}{r_A}$$

- Gives the change in potential when going from $A \rightarrow B$, as $r_B \rightarrow \infty$

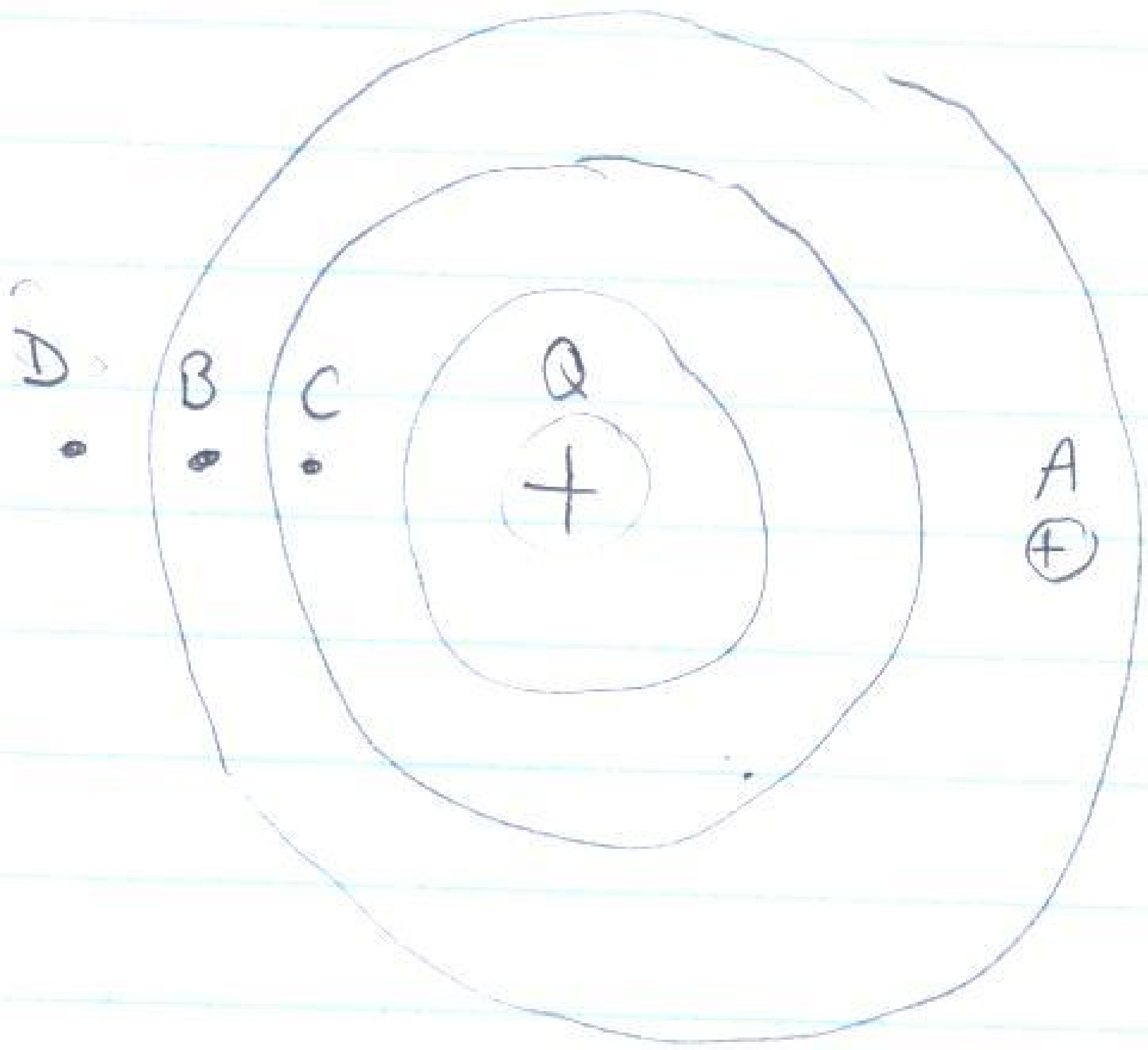
- define this to $V_B = 0$

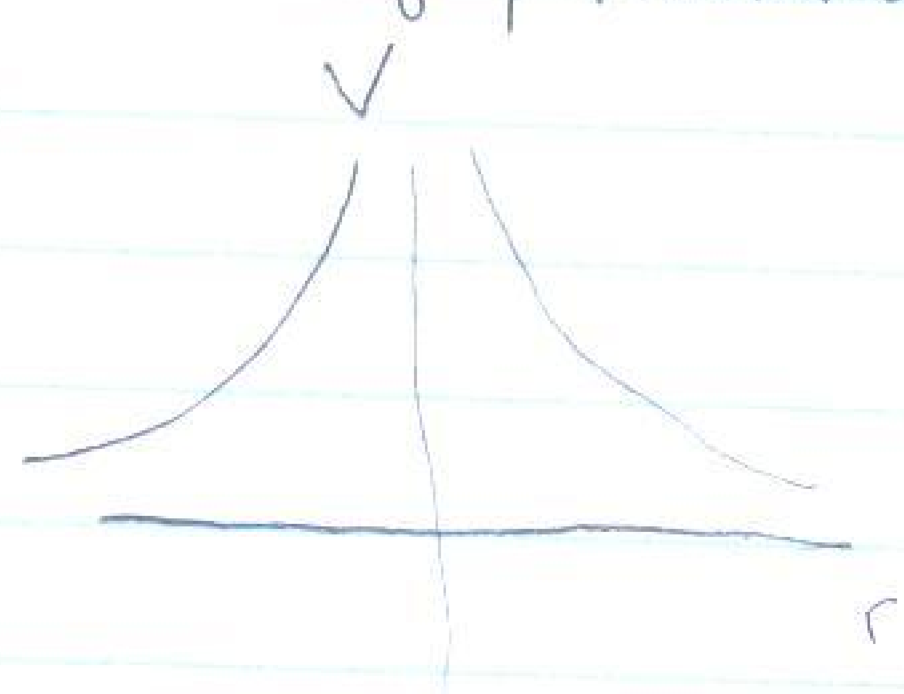
$$\boxed{V_A = k_e Q / r_A}$$

Physletts

http://qbx6.ltu.edu/s_schneider/physlets/main/gaus_rings.shtml

equipotentials.shtml





How much work ^by you^ does it take to move from

>0 $\quad$ <0

W_{AC} ✓

W_{AB} $= 0$

W_{AD} ✓

For example:

$$W_{A\to C} = U_C - U_A =$$

$$W_{A\to C} = \frac{k_e Q q}{r_C} - \frac{k_e Q}{r_A}$$

$$W_{A\to C} = k_e Q q \left(\frac{1}{r_C} - \frac{1}{r_A} \right)$$

For, $r_C > r_A$, this is positive

Then consider a bunch of charges:

The total potential is a just a sum:

$$V(r) = \sum_i \frac{k_e Q_i}{|\vec{r} - \vec{r}_i|}$$

qV = the energy required to bring a point charge from in from infinity to poin r

Then

$$V = \sum_i \frac{k_e Q_i}{|\vec{r} - \vec{r}_i|}$$

$$V = \frac{k_e (+q)}{(x + a)} - \frac{k_e q}{(a - x)}$$

$$V = -\frac{2 k_e q \quad x}{(a^2 - x^2)} = -\frac{2 k_e q x}{(a^2 - x^2)}$$

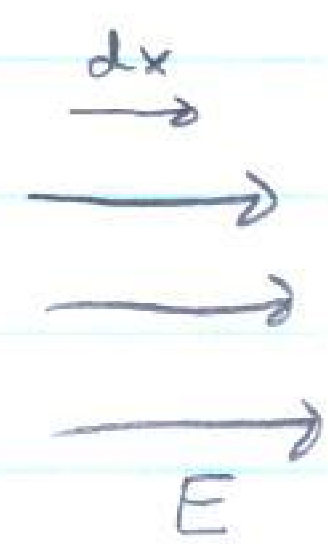
Is there a way to determine the Electric Field from the potential?

First Example

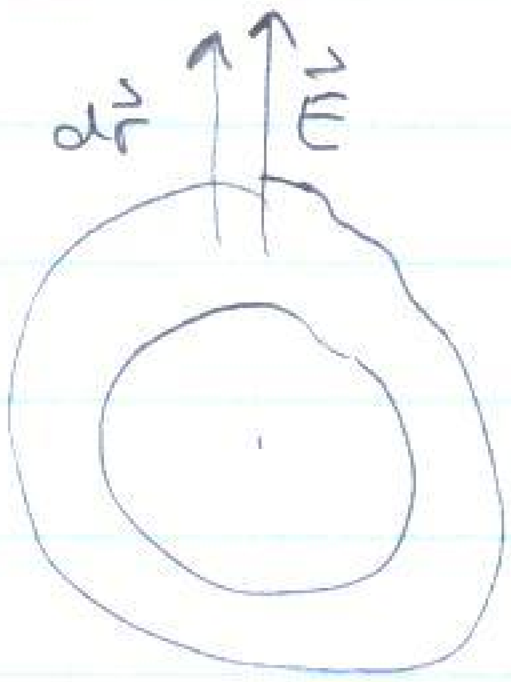
$$dV = -\vec{E} \cdot d\vec{x}$$

Suppose $\vec{E}$ points in x direction then

$$\boxed{E_x = -\frac{dV}{dx}}$$



Second Example For A radial symmet distribution



$$dV = -E_r \, dr$$

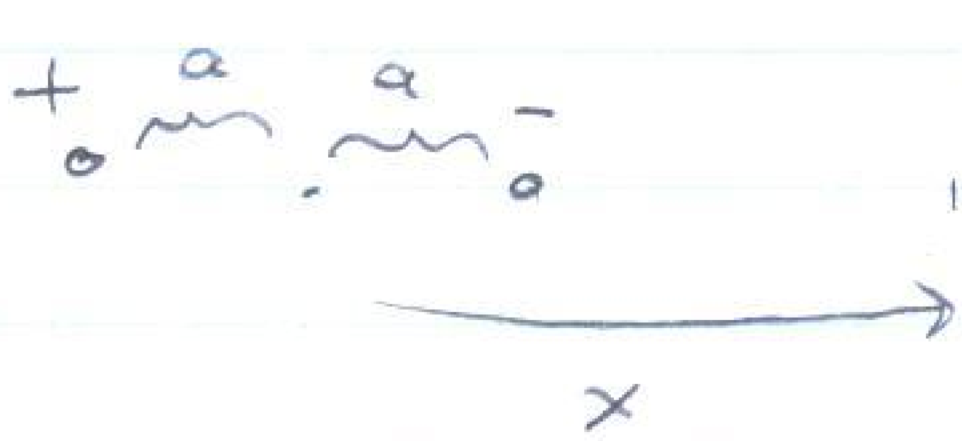
$$E_r = -\frac{dV}{dr}$$

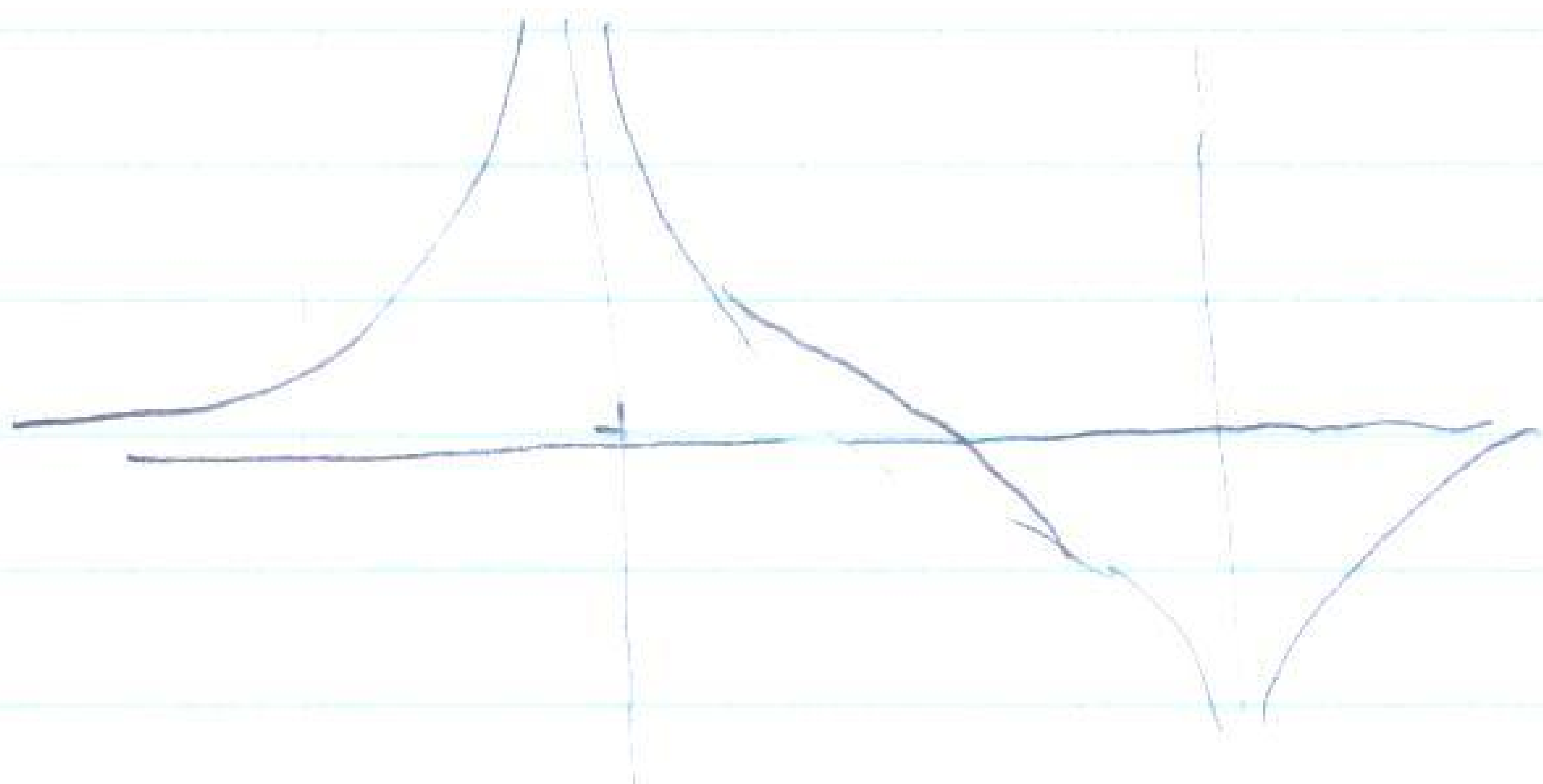
General Case:

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad \text{and} \qquad E_z = -\frac{\partial V}{\partial z}$$

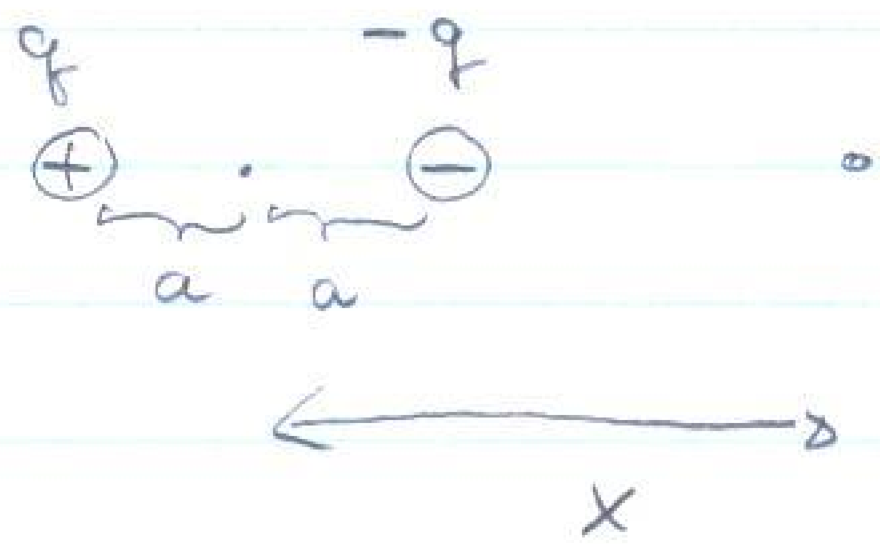
- You can find the electric field completely once you know the potential
- It is usually easier to compute the potential and then differentiate

Problem: Compute electric Field from the dipole shown below





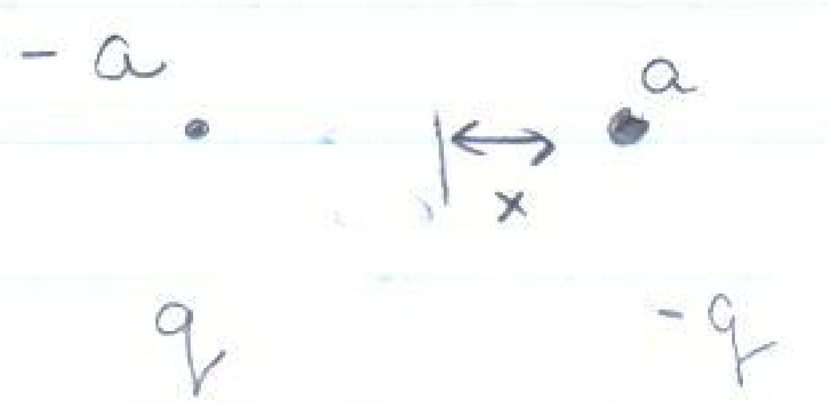
Problem: Given one positive and one negative charge



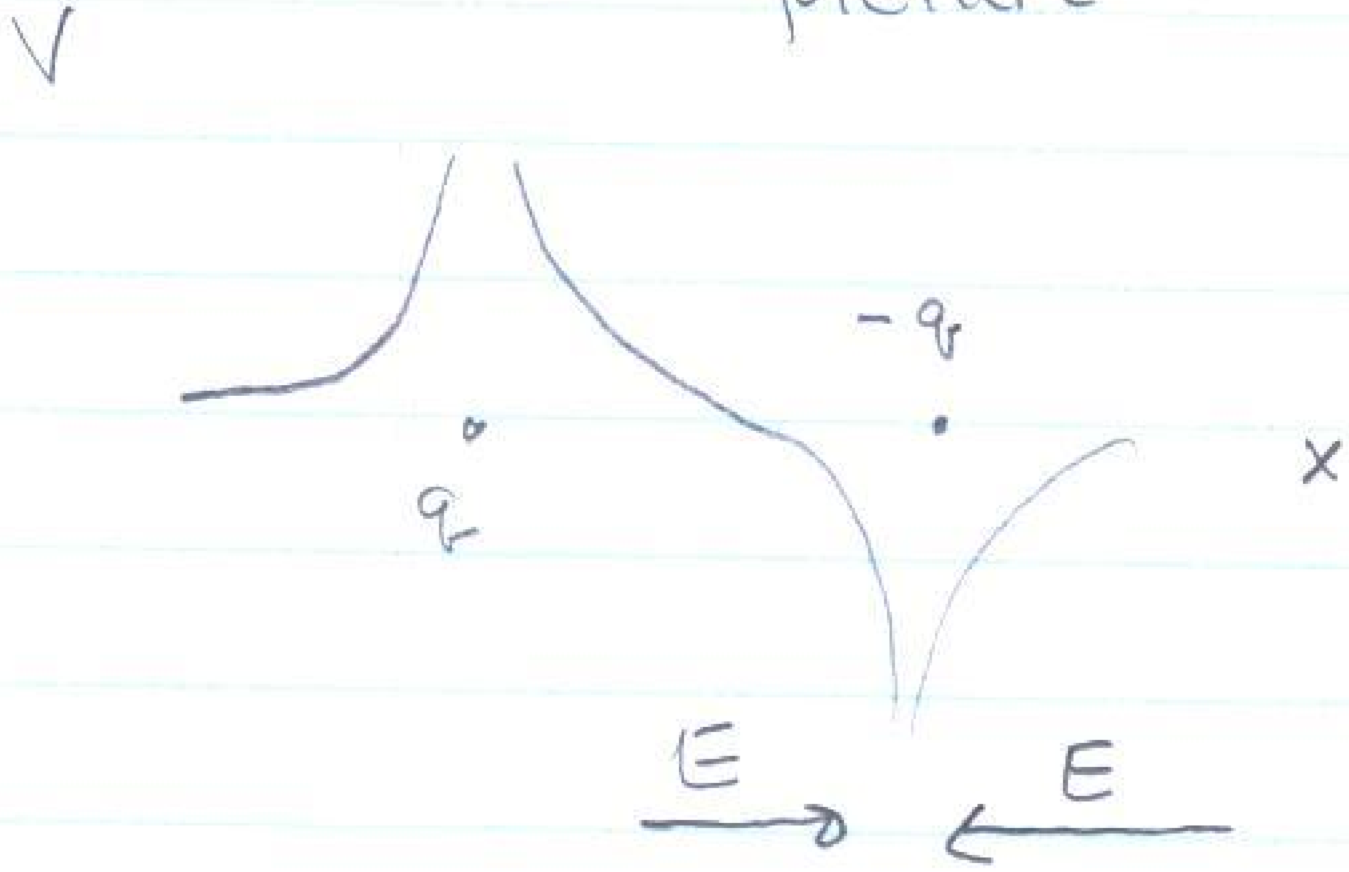
A Determime the potential at a point x

$$V = k_e q\left(\frac{1}{x+a} - \frac{1}{x-a}\right) = -\frac{2k_e q a}{x^2 - a^2}$$

B Suppose we want to know the potential between the two charges



The picture



Field Lines and Equipotential Surface

- Look at applet lines of equi-potential are {orthogonal} to Field lines
  (perpendicular)

$dV = \vec{E} \cdot d\vec{s} = E\, ds \cos\theta = 0$ ← equipontential means
$dV = 0 \quad V = \text{Const}$

- only true if $\cos\theta = 90^\circ$

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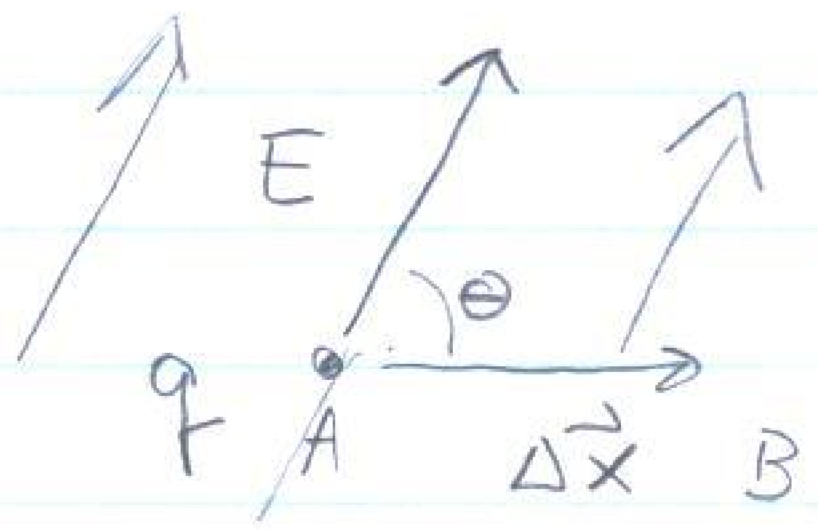

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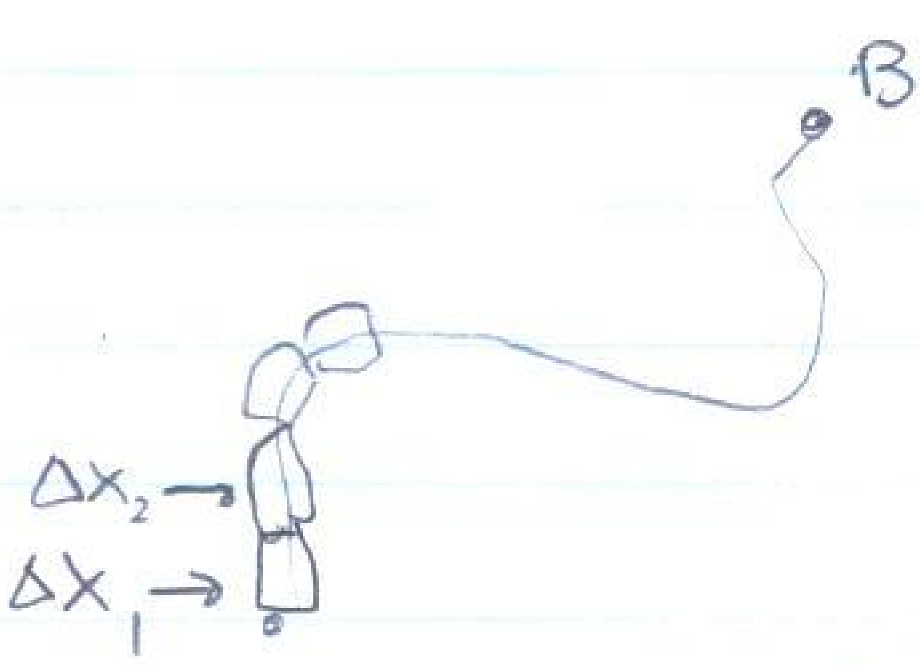

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