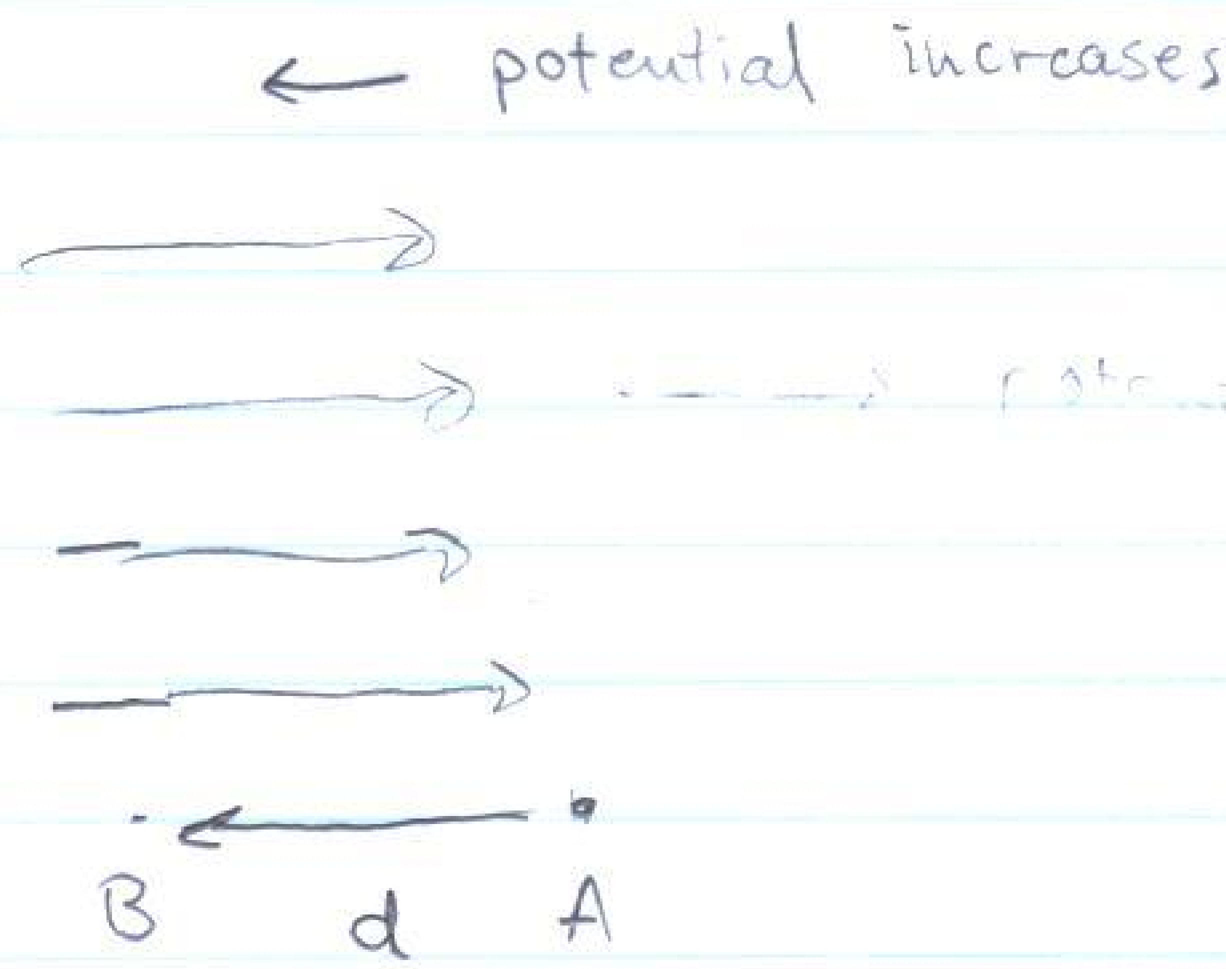
Last Time

Electrostatic Potential

- What is it?

$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{x}$$

$$= -\vec{E} \cdot \vec{d} \quad \text{for a constat field}$$



- Why should I care?

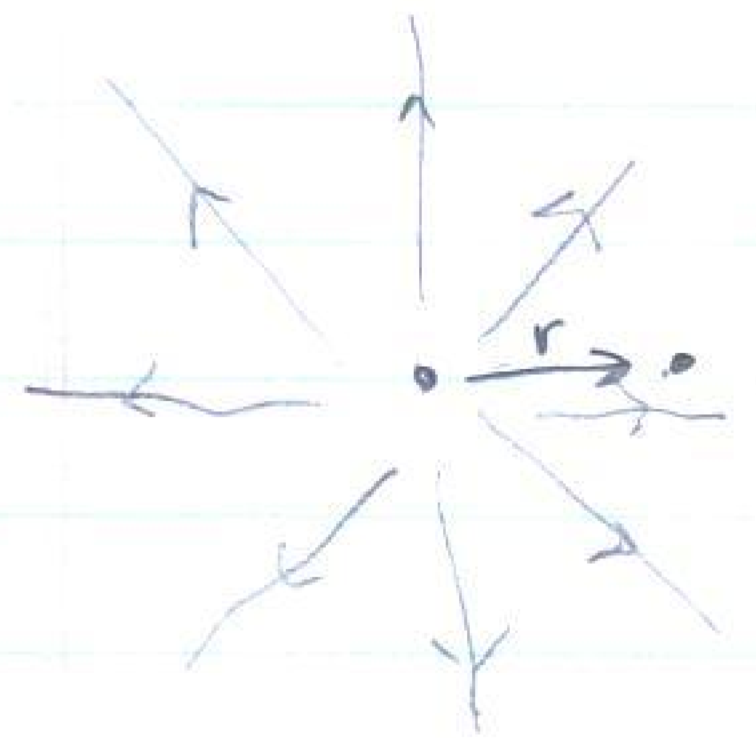
  - Because the world is made up of charges

The change in Potential E to move a charge from A to B is

$$\underbrace{\Delta U}_{\text{work required to move a charge from A to B}} = q\,\Delta V$$

work required to move a charge from A to B

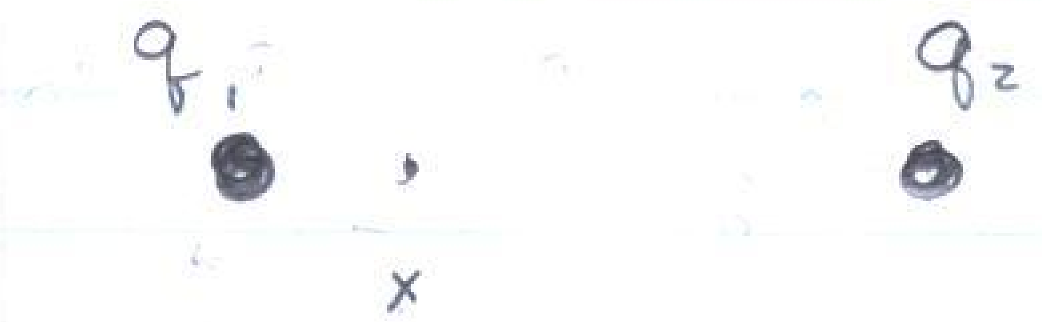
- How do I compute it?



- Work it out for a point charge;

$$V(r) = k_e \frac{Q}{r}$$

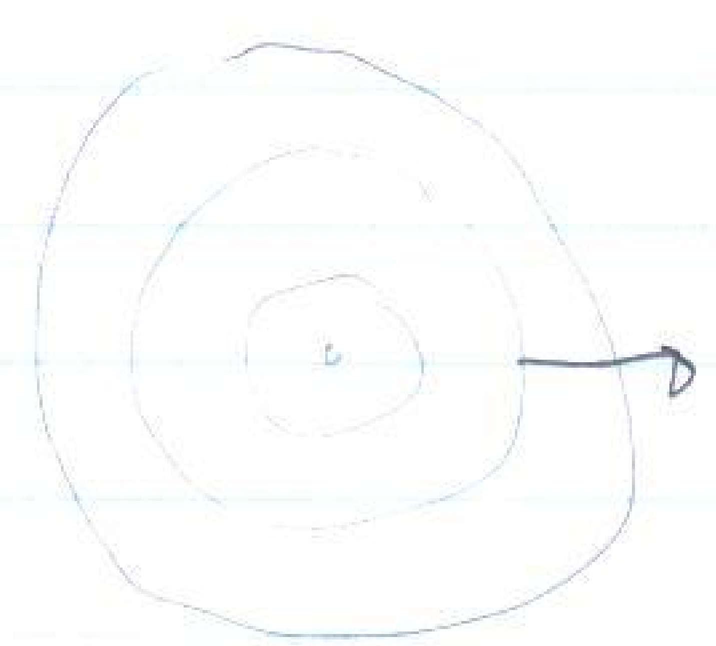
Then the total potential at point r



is just a sum of the potentials from all the charges

- How do I compute the Electric field from it?

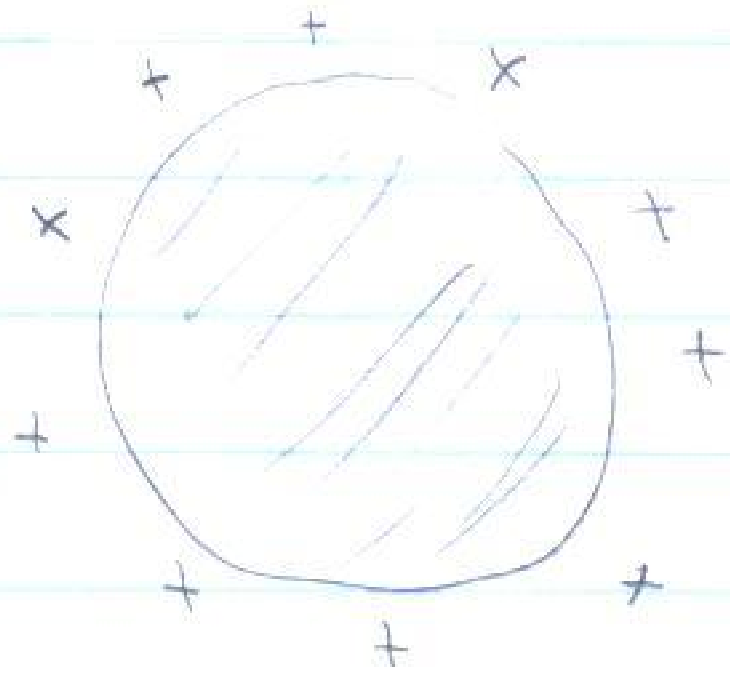
- $$E_x = -\frac{\partial V}{\partial x}$$

- 

$$E_r = -\frac{\partial V}{\partial r}$$

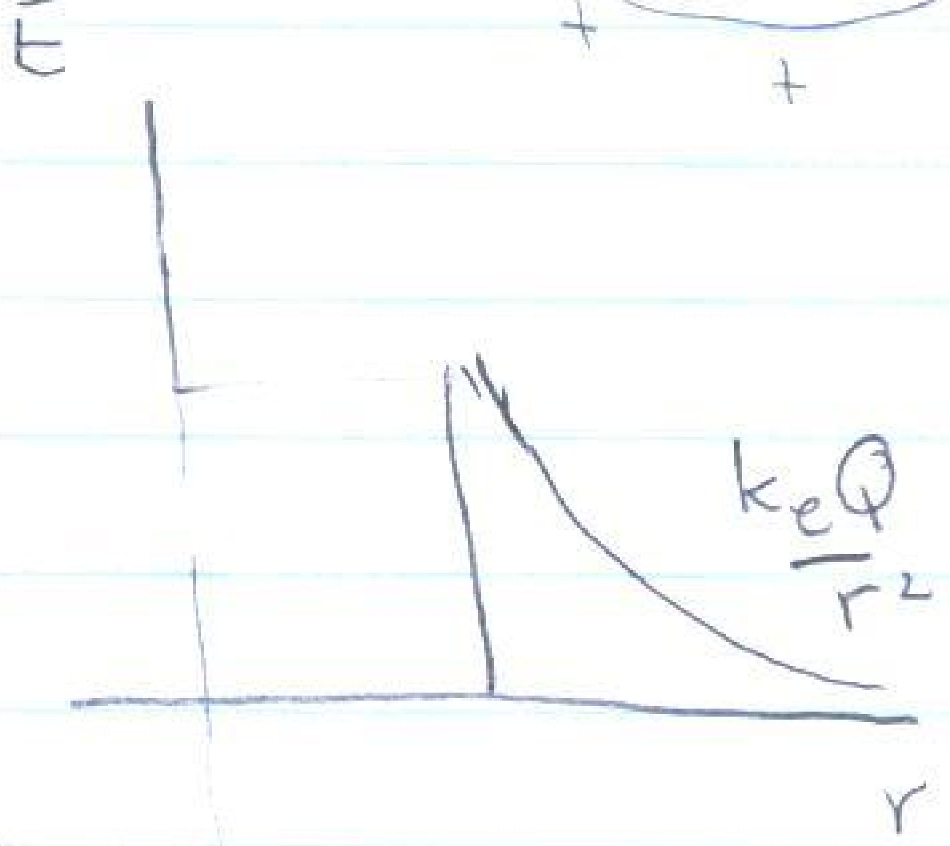
Conductors

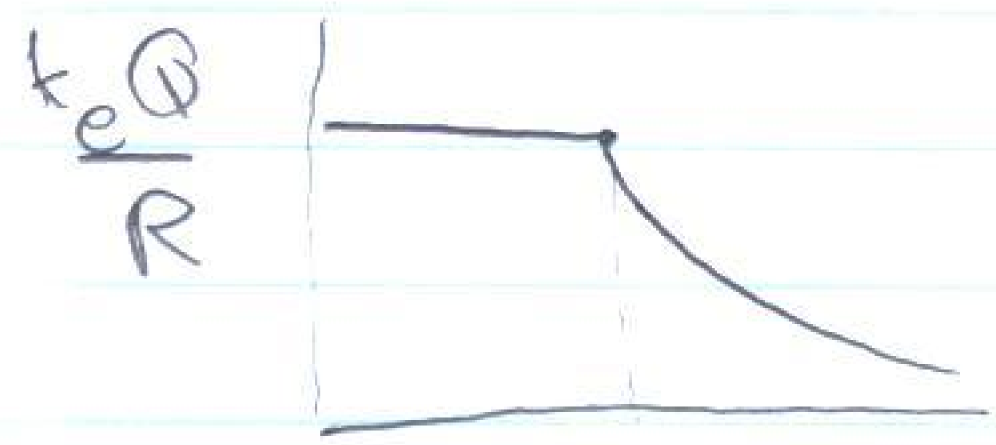
- Electric Field is zero inside a condutor



$$\Delta V = \vec{E} \cdot \vec{d} = 0$$

- Potential Is Constant inside a conductor

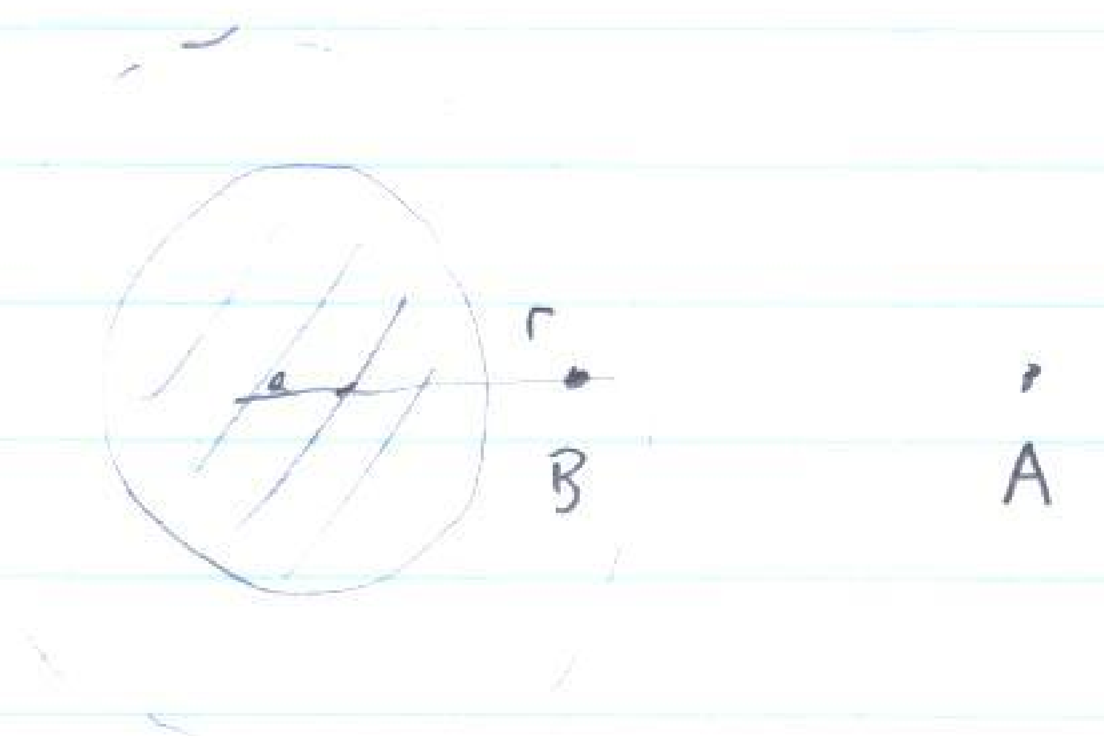




$$V_r = k_e \frac{Q}{r}$$ (For r-outside)

$$V = \frac{k_e Q}{R}$$ ← on surface and inside

Electric Potential inside the Sphere



$$E = \frac{k_e Q}{r^2}$$

$$V_A - V_B = -\int_B^A \vec{E} \cdot d\vec{r} = -\int_B^A \frac{k_e Q}{r^2}\, dr$$

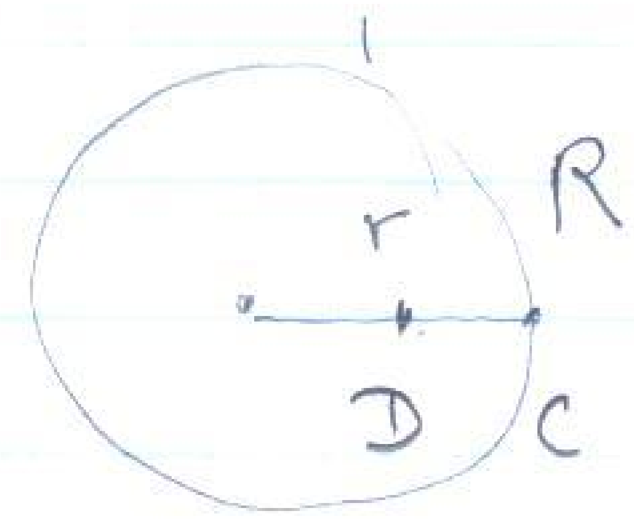
$$= +\frac{k_e Q}{r}\Bigg|_B^A$$

$$\cancelto{0}{V_A} - V_B = \cancelto{0}{\frac{k_e Q}{r_A}} - \frac{k_e Q}{r_B}$$

take $V_A = 0$ at $r_A = \infty$

$$V_B = \frac{k_e Q}{R_B}$$

Now look inside the sphere



Then

$$V_D - V_C = -\int_R^r E_r\, dr = -\int_R^r \frac{k_e Q}{R^3}\, r\, dr$$

$$= -\frac{k_e Q}{R^3}\int_R^r r\, dr = -\frac{k_e Q}{R^3}\,\frac{1}{2}\, r^2 \Big|_R^r$$

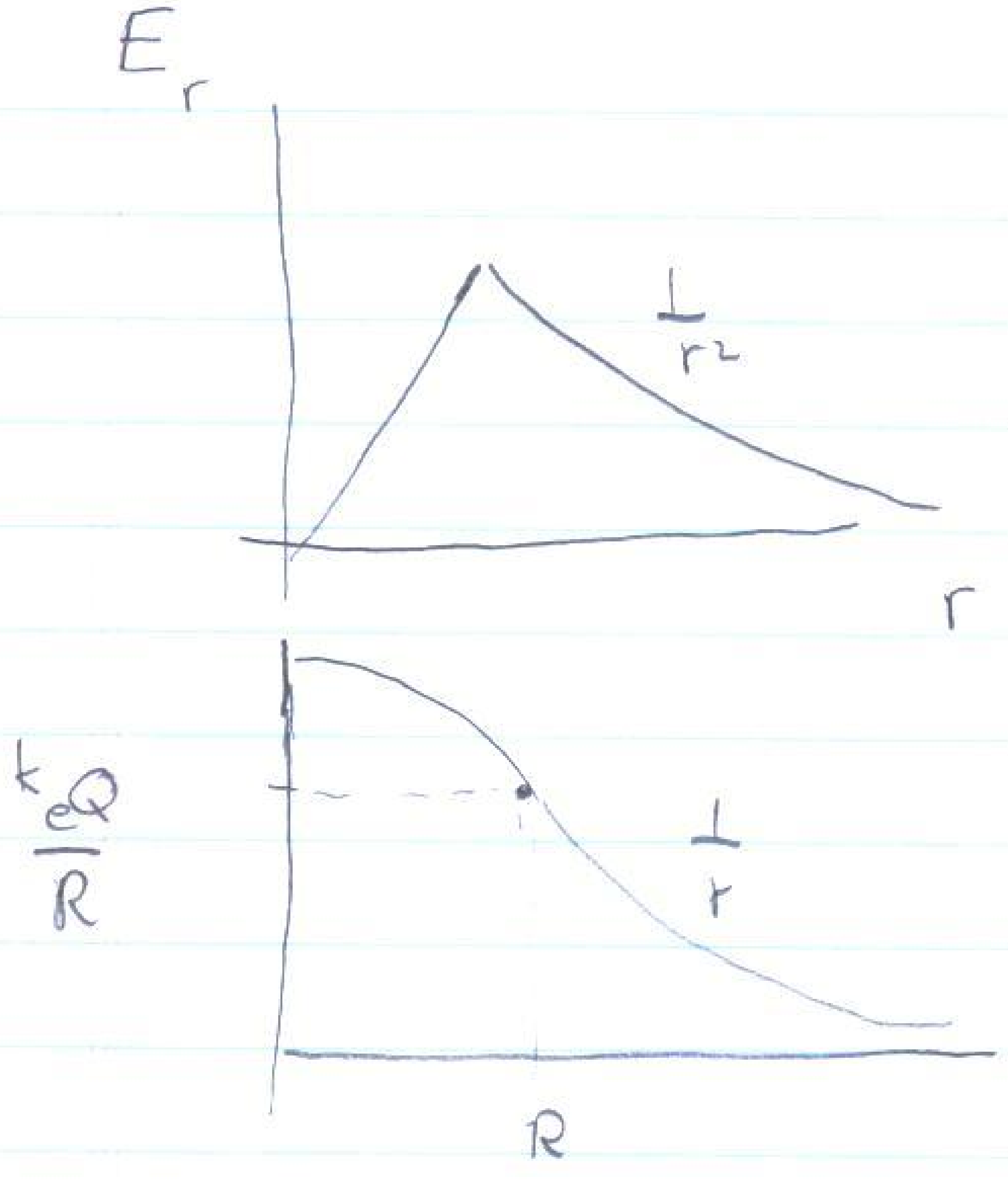
$$= -\frac{k_e Q}{2 \cdot R^3}\left(r^2 - R^2\right)$$

$$V_D - V_C = \frac{k_e Q}{2R^3}\left(R^2 - r^2\right)$$

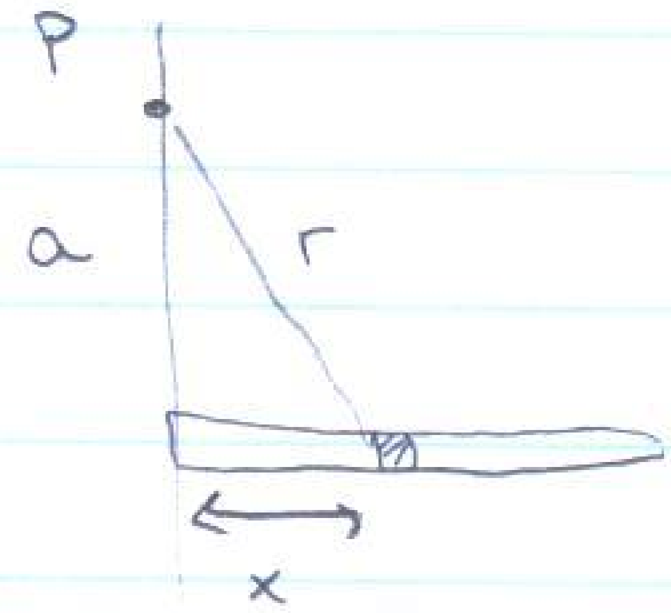
$$V_C = \frac{k_e Q}{R} \Rightarrow V_D = \frac{k_e Q}{2R^3}\left(R^2 - r^2\right) + \frac{k_e Q}{R}$$

$$V_C = \frac{k_e Q}{R}\left(\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right)$$

Result



Equipotential From a line of Charge



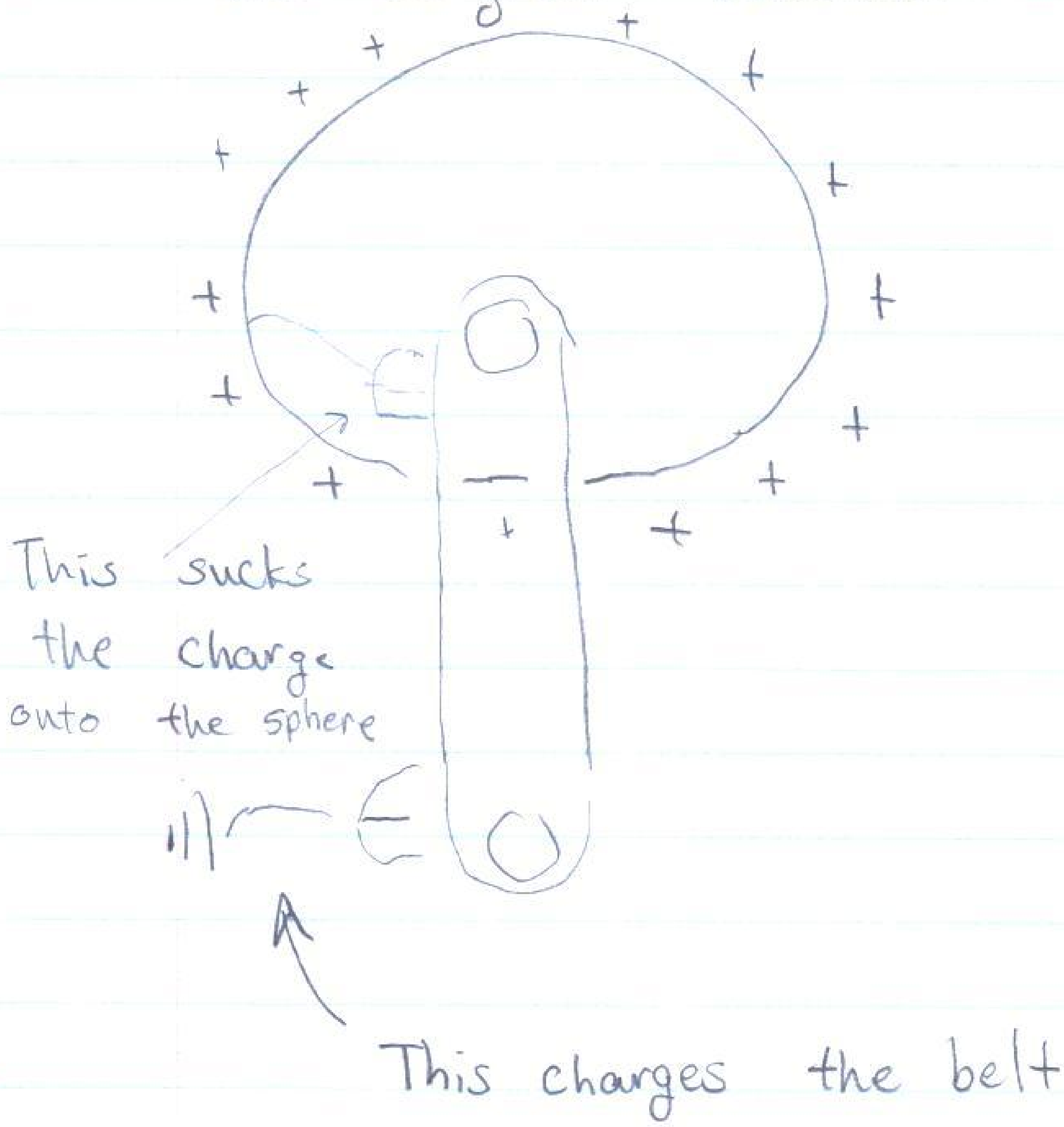
$$dV = k_e \frac{dQ}{r} = \frac{k_e \lambda \, dx}{\sqrt{x^2 + a^2}}$$

$$V = \int_0^L \frac{k_e \lambda \, dx}{(x^2 + a^2)^{1/2}} = \frac{k_e Q}{L} \int_0^L \frac{dx}{\sqrt{x^2 + a^2}}$$

$$= \frac{k_e Q}{L} \ln \left(x + \sqrt{x^2 + a^2} \right) \Big|_0^L$$

$$V_e = \frac{k_e Q}{L} \ln \left(\frac{L + \sqrt{L^2 + a^2}}{a} \right)$$

Van-de graff Generator



E·d

potential increases for a constat field

d A

d I care?

world is made up of charges

Potential E to move a charge

is ΔV

a charge from A to B