Coulomb law
\[ \mathbf{F} = k_e \frac{Q_1 Q_2}{r^2}, \hat{r} \] (1)
\[ k_e = \frac{1}{4\pi\epsilon_0} = 8.98 \times 10^9 \text{Nm}^2/\text{C}^2 \] (2)
\[ \epsilon_0 = 8.85 \times 10^{-12} \text{C}^2/(\text{Nm}^2) \] (3)
\[ \mathbf{E} = \frac{\mathbf{F}}{q_o} \] (4)
For a point charge
\[ \mathbf{E} = k_e \frac{Q}{r^2} \hat{r} \] (5)
For field lines
\[ E \propto \frac{\text{# of lines}}{\text{Area}} \] (6)
\[ a = \frac{q E}{m} \] (7)
Gauss Law
\[ \Phi_E = \int \mathbf{E} \cdot d\mathbf{A} \] (8)
\[ \Phi_E = \frac{\mathbf{E} \cdot \mathbf{A}}{4\pi} \text{ for constant field} \] (9)
\[ \Phi_E = EA \cos(\theta) \text{ for constant field} \] (10)
\[ \Phi_E = 4\pi k_e Q_{\text{net}} \] (11)
\[ \Phi_E = \frac{Q_{\text{net}}}{\epsilon_0} \] (12)
1. Uniformly Charged slab.
\[ E = \frac{\sigma}{2\epsilon_0} \] (13)
2. Two charged slabs
\[ E = \begin{cases} \frac{\sigma}{\epsilon_0} & \text{Inbetween the plates} \\ 0 & \text{Outside the plates} \end{cases} \] (14)
3. The electric field from a long line of charge
\[ E_r = \frac{2k_e \lambda}{r} \] (15)
4. The electric field from a uniformly charged insulating sphere with total charge \( Q \) and radius \( R \),
\[ E_r = \begin{cases} \frac{k_e Q}{R^2} & \text{for } r > R \\ \frac{k_e Q}{r} \frac{r}{R} & \text{for } r < R \end{cases} \] (16)
5. The electric field from a uniformly charged conducting sphere with total charge \( Q \) and radius \( R \),
\[ E_r = \begin{cases} \frac{k_e Q}{r} & \text{for } r > R \\ 0 & \text{for } r < R \end{cases} \] (17)
\[ \Delta V = V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{x} \] (18)
For a constant electric field \( E_z \) in the \( z \) direction
\[ \Delta V = -E_z \cdot d \cos(\theta) \] (19)
\[ \Delta V = -E_z z \] (20)
\[ W_{\text{you}} = U_B - U_A = q(V_B - V_A) \] (22)
or
\[ \Delta U = q \Delta V \] (23)
For a point charge
\[ V(r) = \frac{k_e Q}{r} \] (24)
\[ W_{\text{you}} = U = k_e \frac{q_1 q_2}{r_{12}} + k_e \frac{q_2 q_3}{r_{23}} + k_e \frac{q_1 q_3}{r_{13}} \] (25)
\[ E_z = -\frac{\partial V(z)}{\partial z} \] (26)
\[ E_r = -\frac{\partial V(r)}{\partial r} \] (27)
\[ E_z = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z} \] (28)
\[ V = \sum k_e \frac{\Delta q}{r} = \int k_e \frac{dq}{r} \] (29)
1. Uniformly charged insulating sphere with charge \( Q \) and radius \( R \)
\[ V(r) = \begin{cases} \frac{k_e Q}{2R} (3 - r^2/R) & \text{for } r > R \\ \frac{k_e Q}{2R} & \text{for } r < R \end{cases} \] (30)
2. Conducting Sphere with charge \( Q \) and radius \( R \)
\[ V(r) = \begin{cases} \frac{k_e Q}{2R} & \text{for } r > R \\ \frac{k_e Q}{2R} & \text{for } r < R \end{cases} \] (31)
3. The potential of a charged disc of radius \( a \)
\[ V = 2\pi k_e \sigma \left( \sqrt{x^2 + a^2} - x \right) \] (32)
4. For a ring of radius $a$

$$V = k_e \frac{Q}{\sqrt{x^2 + a^2}}$$

(33)

Capacitance

$$C \Delta V = Q$$

(34)

1. For two plates of area $A$ and separation $d$ the capacitance is

$$C = \varepsilon_o \frac{A}{d}$$

(35)

2. For a coaxial cable of length $L$ with inner radius $a$ and outer radius $b$ the capacitance is

$$C = \frac{L}{2k_e \ln(b/a)}$$

with $k_e = 1/(4\pi\varepsilon_o)$

(36)

Parallel:

$$C_{eq} = C_1 + C_2 + C_3 + \ldots$$

(37)

Series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

(38)

Kirchoff Laws

1. For each wire indicate a current with an arrow

2. The sum of the currents entering a vertex is zero. Thus in Fig 4. the sum of the vertex

$$I_1 + (-I_2) + (-I_3) = 0$$

(56)

where we have written $(-I_2)$ and $(-I_3)$ because these currents are drawn exiting rather than entering the vertex.

3. For every closed loop, draw a circle and indicate the loop direction. The sum of the potential drops going around the loop is zero.

$$\sum \Delta V = 0$$

(57)

(a) If the current is moving with loop direction (Fig. 5) the voltage drop across the resistor is

$$(\Delta V)_R = -IR$$

(58)

If the current and loop direction are opposite get $+IR$

Currents and Circuits

$$I = \frac{dQ}{dt} = \text{Charge passing through surface } A$$

$$\frac{\Delta t}{(47)}$$
(b) If the loop direction is the with the battery (Fig. 3) the voltage change is

\[(\Delta V)_E = +E\]

(59)

If the loop and battery are opposite \(-E\)

(c) For each capacitor if the loop direction is in the same as the current direction (Fig. 1) the voltage drop is

\[(\Delta V)_C = -\frac{q}{C}\]

(60)

where \(q\) is the charge on the capacitor and \(C\) is the capacitance. If current and loop direction are opposite get \(+q/C\).

Capacitor Charging

\[q(t) = Q(1 - e^{-\frac{t}{\tau}})\]

(61)

\[I(t) = \frac{\dot{E}}{R}e^{-\frac{t}{\tau}}\]

(62)

with

- \(Q = CE\).
- \(I_o = E/R\).
- \(\tau = RC\)

Capacitor Discharging

\[q(t) = Qe^{-\frac{t}{\tau}}\]

(64)

\[I(t) = -I_o e^{-\frac{t}{\tau}}\]

(65)

with \(I_o = Q/RC\).

### Forces and Magnetic Fields

\[\mathbf{F} = q\mathbf{v} \times \mathbf{B}\]

(66)

\[r = \frac{mv}{qB}\]

(67)

\[d\mathbf{F} = Id\mathbf{L} \times \mathbf{B}\]

(68)

For a uniform magnetic field

\[\mathbf{F} = I\mathbf{L}_{CD} \times \mathbf{B}\]

(69)

where \(\mathbf{L}_{CD}\) is the line connecting \(C\) to \(D\). A corollary is that a closed loop in a uniform magnetic field experiences no net force (it does experience a torque though).

\[= I\mathbf{A}\]

(70)

\[\tau = \mu \times \mathbf{B}\]

(71)

\[U = -\mu \cdot \mathbf{B} = -\mu B \cos(\theta)\]

(72)
\[ \Delta V = -\frac{q}{c} \quad \text{Fig. 1} \]

\[ \Delta V = \varepsilon \quad \text{Fig. 2} \]

\[ \Delta V = -\frac{q}{c} \quad \text{Fig. 3} \]

\[ I_1 + (-I_2) + (-I_3) = 0 \quad \text{Fig. 4} \]

\[ (\Delta V) = -IR \quad \text{Fig. 5} \]