Coulomb law

$$\mathbf{F} = k_e \frac{Q_1 Q_2}{r^2}, \hat{\mathbf{r}} \tag{1}$$

$$k_e = \frac{1}{4\pi\epsilon_o} = 8.98 \times 10^9 \, \mathrm{Nm^2/C^2} \tag{2}$$

$$\epsilon_o = 8.85 \times 10^{-12} C^2/(Nm^2) \tag{3}$$

$$\mathbf{E} = \frac{\mathbf{F}}{q_o} \tag{4}$$

For a point charge

$$\mathbf{E} = k_e \frac{Q}{r^2} \hat{\mathbf{r}} \tag{5}$$

For field lines

$$E \propto \frac{\text{\# of lines}}{\text{Area}} \tag{6}$$

$$\mathbf{a} = \frac{q\mathbf{E}}{m} \tag{7}$$

Gauss Law

$$\begin{aligned} \Phi_E &= \int \mathbf{E} \cdot d\mathbf{A} && (8) \\ &= \mathbf{E} \cdot \mathbf{A} \quad \text{for constant field} && (9) \\ &= EA\,cos(\theta) \quad \text{for constant field} && (10) \end{aligned}$$

$$\begin{aligned} \Phi_E &= 4\pi k_e Q_{\text{net}} && (11) \\ &= \frac{Q_{\text{net}}}{\epsilon_o} && (12) \end{aligned}$$

1. Uniformly Charged slab.

$$E = \frac{\sigma}{2\epsilon_o} \tag{13}$$

2. Two charged slabs

$$E = \begin{cases} \frac{\sigma}{\epsilon_o} & \text{Inbetween the plates} \\ 0 & \text{Outside the plates} \end{cases} \tag{14}$$

3. The electric field from a long line of charge

$$E_r = \frac{2k_e\lambda}{r} \tag{15}$$

4. The electric field from a uniformly charged insulating sphere with total charge Q and radius R,

$$E_r = \begin{cases} \frac{k_e Q}{r^2} & \text{for } r > R \\ \frac{k_e Q}{R^2}\,\frac{r}{R} & \text{for } r < R \end{cases} \tag{16}$$

5. The electric field from a uniformly charged conducting sphere with total charge Q and radius R,

$$E_r = \begin{cases} \frac{k_e Q}{r^2} & \text{for } r > R \\ 0 & \text{for } r < R \end{cases} \tag{17}$$

$$\Delta V = V_B - V_A \;=\; -\int_A^B \mathbf{E} \cdot d\mathbf{x} \tag{18}$$

For a constant electric field E_z in the z direction

$$\begin{aligned} \Delta V &= -\mathbf{E} \cdot \mathbf{d} && (19) \\ &= -E_z\, d\, cos(\theta) && (20) \\ &= -E_z\, z && (21) \end{aligned}$$

$$W^{\text{you}}_{AB} = U_B - U_A = q(V_B - V_A) \tag{22}$$

or

$$\Delta U = q\,\Delta V \tag{23}$$

For a point charge

$$V(r) = \frac{k_e Q}{r} \tag{24}$$

$$W^{\text{you}} = U = k_e \frac{q_1 q_2}{r_{12}} + k_e \frac{q_2 q_3}{r_{23}} + k_e \frac{q_1 q_3}{r_{13}} \tag{25}$$

$$E_z = -\frac{\partial V(z)}{\partial z} \tag{26}$$

$$E_r = -\frac{\partial V(r)}{\partial r} \tag{27}$$

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z} \tag{28}$$

$$V = \sum k_e \frac{\Delta q}{r} = \int k_e \frac{dq}{r} \tag{29}$$

1. Uniformly charged insulating sphere with charge Q and radius R

$$V(r) = \begin{cases} \frac{k_e Q}{r} & \text{for } r > R \\ \frac{k_e Q}{2R}\left(3 - r^2/R\right) & \text{for } r < R \end{cases} \tag{30}$$

2. Conducting Sphere with charge Q and radius R

$$V(r) = \begin{cases} \frac{k_e Q}{r} & \text{for } r > R \\ \frac{k_e Q}{R} & \text{for } r < R \end{cases} \tag{31}$$

3. The potential of a charged disc of radius a

$$V = 2\pi k_e \sigma \left(\sqrt{x^2 + a^2} - x\right) \tag{32}$$

4. For a ring of radius a

$$V = k_e \frac{Q}{\sqrt{x^2 + a^2}} \tag{33}$$

Capacitance

$$C\Delta V = Q \tag{34}$$

1. For two plates of area A and separation d the capacitance is

$$C = \epsilon_o \frac{A}{d} \tag{35}$$

2. For a coaxial cable of length L with inner radius a and outer radius b the Capacitance is

$$C = \frac{L}{2k_e \ln(b/a)} \tag{36}$$

with $k_e = 1/(4\pi\epsilon_o)$

Parallel:

$$C_{\rm eq} = C_1 + C_2 + C_3 + \dots \tag{37}$$

Series:

$$\frac{1}{C_{\rm eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \tag{38}$$

$$U = \frac{1}{2} C(\Delta V)^2 = \frac{Q^2}{2C} \tag{39}$$

$$\frac{U}{\rm Vol} = \frac{1}{2}\epsilon_o E^2 \tag{40}$$

$$E = \frac{E_o}{\kappa} \tag{41}$$

$$\sigma_{\rm ind} = \left(1 - \frac{1}{\kappa}\right)\sigma_o \tag{42}$$

$$C = \kappa C_o \tag{43}$$

$$p \equiv 2aq \tag{44}$$

$$\tau = \mathbf{p} \times \mathbf{E} \tag{45}$$

$$U = -\mathbf{p} \cdot \mathbf{E} \tag{46}$$

Currents and Circuits

$$I = \frac{dQ}{dt} = \frac{\text{Charge passing through surface } A}{\Delta t} \tag{47}$$

$$V = IR \tag{48}$$

$$\mathscr{P} = I^2 R \tag{49}$$

$$\mathbf{J} = \frac{I}{A} = nq\mathbf{v}_d \tag{50}$$

$$\mathbf{J} = \sigma \mathbf{E} \tag{51}$$

$$\rho = \frac{1}{\sigma} \tag{52}$$

$$R = \rho \frac{\ell}{A} \tag{53}$$

Series

$$R_{eq} = R_1 + R_2 \dots \tag{54}$$

Resistors in $\|$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \dots \tag{55}$$

Kirchoff Laws

1. For each wire indicate a current with an arrow
2. The sum of the currents entering a vertex is zero. Thus in Fig 4. the sum of the vertex

$$I_1 + (-I_2) + (-I_3) = 0 \tag{56}$$

where we have written $(-I_2)$ and $(-I_3)$ because these currents are drawn exiting rather than entering the vertex.

3. For every closed loop, draw a circle and indicate the loop direction. The sum of the potential drops going around the loop is zero.

$$\sum \Delta V = 0 \tag{57}$$

(a) If the current is moving with loop direction (Fig. 5) the voltage drop across the resistor is

$$(\Delta V)_{\rm R} = -IR \tag{58}$$

If the current and loop direction are opposite get $+IR$

(b) If the loop direction is the with the battery (Fig. 3) the voltage change is

$$(\Delta V)_{\mathscr{E}} = +\mathscr{E} \tag{59}$$

If the loop and battery are opposite $-\mathscr{E}$

(c) For each capacitor if the loop direction is in the same as the current direction (Fig. 1) the voltage drop is

$$(\Delta V)_C = -\frac{q}{C} \tag{60}$$

where q is the charge on the capacitor and C is the capacitance. If current and loop direction are opposite get $+q/C$.

Capacitor Charging

$$q(t) = Q\,(1 - e^{-\frac{t}{RC}}) \tag{61}$$

$$I(t) = \frac{\mathscr{E}}{R} e^{-\frac{t}{RC}} \tag{62}$$

with

- $Q = C\mathscr{E}$.
- $I_o = \mathscr{E}/R$.

$$\tau = RC \tag{63}$$

Capacitor Discharging

$$q(t) = Qe^{-\frac{t}{RC}} \tag{64}$$

$$I(t) = -I_o e^{-\frac{t}{RC}} \tag{65}$$

with $I_o = Q/RC$.

Forces and Magnetic Fields

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \tag{66}$$

$$r = \frac{mv}{qB} \tag{67}$$

$$d\mathbf{F} = Id\mathbf{L} \times \mathbf{B} \tag{68}$$

For a uniform magnetic field

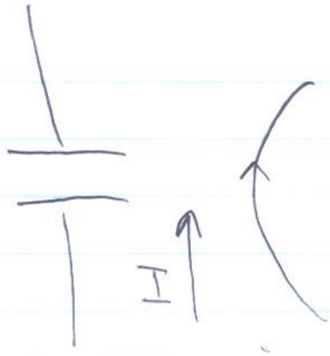
$$\mathbf{F} = I\mathbf{L}_{CD} \times \mathbf{B} \tag{69}$$

where $\mathbf{L}_{CD}$ is the line connecting C to D. A corrlary is that a closed loop in a uniform magnetic field experiences no net force (it does experience a torque though).

$$= I\mathbf{A} \tag{70}$$

$$\tau = \mu \times \mathbf{B} \tag{71}$$

$$U = -\mu \cdot \mathbf{B} = -\mu B \cos(\theta) \tag{72}$$

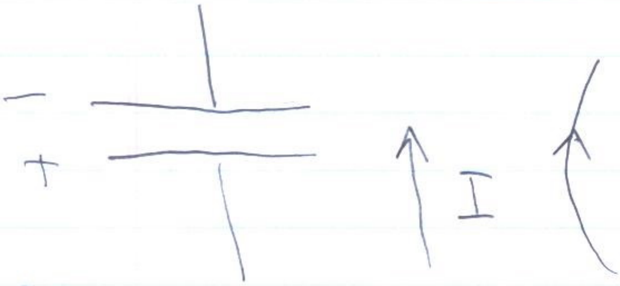


$\Delta V = -\frac{q}{C}$

Fig 1

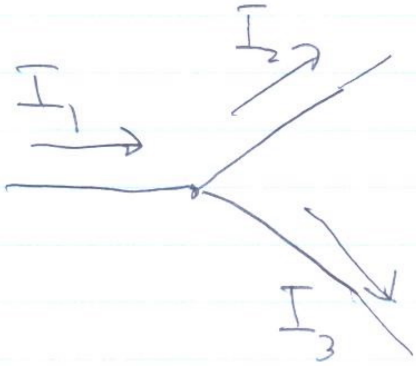
$\Delta V = \mathcal{E}$

Fig 2



$\Delta V = -\frac{q}{C}$

Fig3



$I_1 + (-I_2) + (-I_3) = 0$

Fig 4

$(\Delta V) = -IR$

Fig. 5