Problems

34.5, 34.7, 34.15, 34.17, 34.31, 32.5, 32.7, 32.11, 32.53, 32.47, 32.51

Students wishing to not take the final exam must turn in the underlined problems to me before Wednessday. If the quality of the homework is not sufficiently high they will not be exempt.

Inductance

- For a typical coil, the current in changing, the resulting B field is changing, the changing B-field produces a changing magnetic flux, and a changing magnetic flux produces a voltage known as the back emf

$$\mathcal{E} = -L\frac{dI}{dt} \tag{1}$$

  The inducance in any coil is

$$L = \frac{N\Phi_B}{I} \tag{2}$$

  where Φ_B is the magnetic flux through the coil for a fixed current and N is the number of turns.

- For an ideal solenoid the inductance is

$$L = \frac{\mu_o N^2 A}{\ell} \tag{3}$$

  Here N is the the number of turns, A is the cross sectional area, ℓ is the length of solenoid. You should be able to derive this result.

- The Energy stored in an inductor is

$$U = \frac{1}{2}LI^2 \tag{4}$$

  The energy is stored within the magnetic field. The energy per unit volume is

$$u_B = \frac{B^2}{2\mu_o} \tag{5}$$

- LC circuits
  - In an L, C circuit that has zero resistance the current and charge on the capacitor change as

$$Q = Q_{\max}\cos(\omega_o t + \phi) \tag{6}$$

$$I = \frac{dQ}{dt} = -\underbrace{\omega_o Q_{\max}}_{I_{\max}}\sin(\omega_o t + \phi) \tag{7}$$

    where $Q_{\max}$ is the maximum charge on the capacitor and

$$\omega_o = \frac{1}{\sqrt{LC}} \tag{8}$$

    is the oscillation frequency of the circuit
  - The energy in the LC circuit is constant and is the energy stored in the

$$U = \frac{1}{2}LI^2(t) + \frac{1}{2}\frac{Q^2(t)}{C} \tag{9}$$

    Note at certain moments there is no current and only charge. At other moments there is only current and no charge, so

$$U = \frac{1}{2}LI^2_{\max} = \frac{1}{2}\frac{Q^2_{\max}}{C} \tag{10}$$

Waves

- In free space the electric field and magnetic field obey the wave equation.

$$\frac{\partial^2 E}{\partial x^2} = \mu_o \epsilon_o \frac{\partial^E}{\partial t^2} \tag{11}$$

$$\frac{\partial^2 B}{\partial x^2} = \mu_o \epsilon_o \frac{\partial^B}{\partial t^2} \tag{12}$$

- The solution to this equation is

$$E = E_{\max} \cos(kx - \omega t) \tag{13}$$

$$B = B_{\max} \cos(kx - \omega t) \tag{14}$$

with

$$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f \qquad c = \lambda f \tag{15}$$

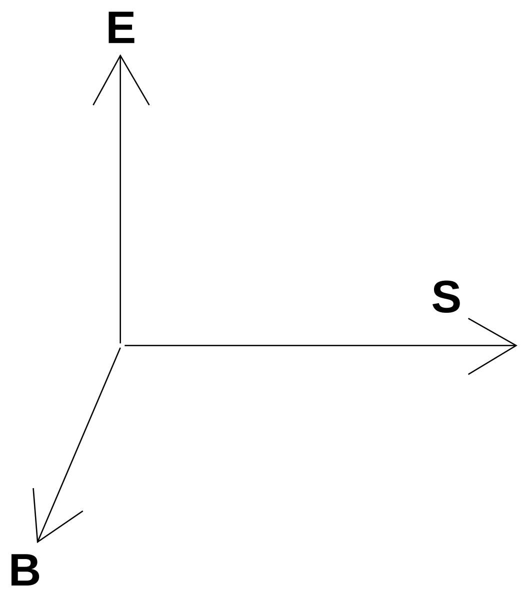
- The magnitude of the electric field and the magnitude of the magnetic field are related

$$E = cB \tag{16}$$

- The waves travel with the speed of light c where

$$c = \frac{1}{\sqrt{\mu_o \epsilon_o}} \tag{17}$$

- Electric field and magnetic field are perpendicular to the direction of propogation. If you take your right hand and curl your fingers from **E** to **B** your thumb points in the direction of propogation. Below **S** points in the direction of propogation.



- The magnetic energy per volume stored in the wave is the same as the electric energy per volume stored in the wave

$$u_E = \frac{1}{2}\epsilon_o E^2 \quad = \quad u_B = \frac{B^2}{2\mu_o} \tag{18}$$

The total energy per volume in the wave is

$$u = u_E + u_B \tag{19}$$

- The energy crossing a surface per unit area per unit time is given by the Poyting vector

$$\mathbf{S} = \frac{1}{\mu_o}\mathbf{E} \times \mathbf{B} \tag{20}$$

  The direction of the Poyting vector is in the direction of propogation.

- Usually the amplitude of the light is oscillating very fast and over short distances, nano-meters and $10^{15}Hz$ for visible light. It therefore makes sense to define the average pointing vector and average energy density etc.

  - For instance

$$\langle u_E \rangle = \left\langle \frac{1}{2}\epsilon_o E^2(t) \right\rangle = \left\langle \frac{1}{2}\epsilon_o E^2_{\max} \cos^2(kx - \omega t) \right\rangle = \frac{1}{4}\epsilon_o E^2_{\max} \tag{21}$$

  - Similarly

$$\langle u_B \rangle = \frac{1}{4\mu_o} B^2_{\max} \tag{22}$$

  - The average magnetic and average electric energies are equal

$$\langle u_B \rangle = \langle u_E \rangle \tag{23}$$

  - The total energy per unit volume is

$$\langle u_{\rm tot} \rangle = \langle u_E + u_B \rangle = \frac{1}{2}\epsilon_o E^2_{\max} = \frac{1}{2\mu_o} B^2_{\max} \tag{24}$$

  - The average rate of energy flow per unit area per unit time is

$$\langle S \rangle = \langle u_{\rm tot} \rangle \, c \tag{25}$$

$$= \frac{E_{\max} B_{\max}}{2\mu_o} \tag{26}$$

  - The momentum per area per time (also known as pressure) carried by the light wave is

$$P = \frac{S}{c} \tag{27}$$

    If the light is completely absorbed then this amount of momentum per area per per time is transfrered to the absorbing object. If the light is totally reflected then the light comes in with this amount of momentum and goes back with this amount of momentum the amount of momentum change per area per time is

$$P = 2\frac{S}{c} \tag{28}$$