E+M in integral form - Qualitative discussion

In this section you should understand the following:

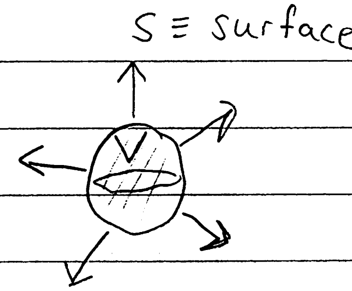
1. In E+M we specify the currents (or a constituent relation) and solve for the fields.

2. We will write down the Maxwell equations in integral form, and describe each term qualitatively

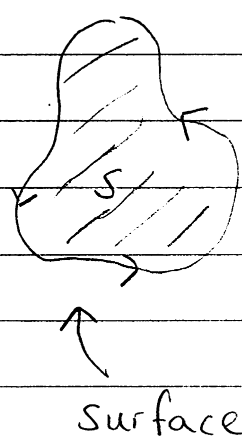
Equations in integral form

Stokes theorem:

(1) $$\int_{\text{Volume}} \nabla \cdot \vec{V} = \int_S d\vec{S} \cdot \vec{V}$$



(2)



$$\int_S d\vec{S} \cdot (\underbrace{\vec{\nabla} \times \vec{V}}_{\text{vector}}) = \int_{\text{loop}} \vec{V} \cdot d\vec{l}$$

Define:

$$\Phi_B \equiv \int_S \vec{B} \cdot d\vec{S} \qquad \Phi_E = \int \vec{E} \cdot d\vec{S}$$

= magnetic flux through surface = electric flux through surface

Then the maxwell equations:

a) $\nabla \cdot E = \rho$

b) $\nabla \times B = \vec{j}/c + {}^1\!/_c\, \partial E/\partial t$

c) $\nabla \cdot B = 0$

d) $-\nabla \times E = {}^1\!/_c\, \partial B/\partial t$

Note: $\vec{j}$ is the current density = Charge / (Area · second)

$I \equiv \int \vec{j} \cdot d\vec{S}$ = charge crossing a surface/time.

read (after integrating eqs. (a) (c) over volume and (b) (d) over area).

a) $$\int_S \vec{E} \cdot d\vec{S} = Q_{enc}$$ (Gauss Law)

b) $$\oint \vec{B} \cdot d\vec{\ell} = \frac{I}{c} + \frac{1}{c}\frac{\partial \Phi_E}{\partial t}$$ (Ampere's Law + Maxwell correction)

c) $$\int_S \vec{B} \cdot d\vec{S} = 0$$ (No magnetic charge)

d) $$-\oint \vec{E} \cdot d\vec{\ell} = \frac{1}{c}\frac{\partial \Phi_B}{\partial t}$$ (Faraday Law)

Lenz law

Qualitative Discussion of each term:

a) $Q \Leftrightarrow \vec{E}$ charges make E-fields (Gauss)

b) $\vec{J} \leftrightarrow \vec{B}$ moving charges make $\vec{B}$-fields (Ampère)

d) Changing $\vec{B}$-fields induce changing $\vec{E}$-fields, which tend to oppose the change (Faraday Law + Lenz Law).

- Changing $\vec{B}$-fields are caused by changing currents or <u>accelerating</u> charges

b) Changing $\vec{E}$-fields create changing $\vec{B}$-fields (Maxwell correction), which in turn makes changing $\vec{E}$-fields etc.

This sets off a wave of light, where changing $\vec{E}$ makes changing $\vec{B}$ and vice versa. To get the process started you need to accelerate charges.

A formula which we will derive, is the Larmour formula, for the total Power radiated by an accelerating (non-relativistic) charged particle

Memorize!

$$\boxed{P = \frac{2}{3}\left(\frac{e^2}{4\pi}\right)\frac{a^2}{c^3}}$$

power $\qquad$ (charge)^2 $\qquad$ (acceleration)^2

Solve For the fields! Specify the currents!

- Generally in E+M class we consider two sorts of problems. In the first case, we consider the fields specified, and solve classical equations of motion

$$F = q(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}),$$

for the trajectory of the particle. This is really classical mechanics, not E+M. We won't do it much.

- In the second case, we consider the currents as specified, and solve Maxwell equations

$$\nabla \cdot E = \rho,$$

$$\nabla \times B = j/c + 1/c\, \partial_t E,$$

$$\nabla \cdot B = 0,$$

$$-\nabla \times E = \frac{1}{c}\frac{\partial B}{\partial t},$$

for the fields. Considering how the radiation changes the current trajectory is <u>not</u> part of classical electrodynamics, but it is part of quantum-electrodynamics.

- So when given a problem, you should ask "what are the currents?" otherwise the problem is ill-posed. The specified currents must obey the continuity equation

$$\partial_t \rho + \nabla \cdot \vec{j} = 0$$

or the maxwell equations will have no solution.

- Very often, the currents are not specified directly in media. Rather, the currents are specified by an additional constitutive relation, a relation between the currents and applied fields. This is something in addition to Maxwell Eqs.

Example $\quad \vec{j} = \sigma \vec{E}$ (Ohm's Law) (Metal)

Then we solve:

$$\nabla \times B = \frac{\sigma}{c} E + \frac{1}{c} \partial_t \vec{E}, \quad \text{etc} \qquad \text{(Ampère)}$$

So for each type of medium (dielectric, metal, super-conductor, etc) we have a new set of maxwell-type equations to solve.

Summary: Before trying to solve a problem in E&M. You must know the currents. Either directly, or through a constitutive relation.