Universality in Chiral Hermitian and Chiral Complex Matrix Models

Gernot Akemann

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Abstract

Universality is proven for all correlation functions of both chiral unitary ensembles, the chiral Hermitian and chiral complex matrix model. These are relevant for applicaitons in QCD for an odd or even number of dimensions. Moreover, the multicritical case is studied, where a universal differential equation is derived determining the corresponding kernels.

Strength Functions in Open Chaotic Systems: Application to Photodissociation

Yoram Alhassid

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Abstract

We discuss the statistics of bound-to-continuum strength functions in an open chaotic system characterized by a non-hermitean random Hamiltonian [1,2]. In particular we discuss application to the indirect photodissociation of molecules. In the regime of non-overlapping resonances (weak coupling limit) the spectral autocorrelation function and its time-domain analog – the survival probability – are derived using random matrix theory [2]. The universal autocorrelation function and the probability distribution of the total photodissociation cross-section for an arbitrary coupling strength are derived using supersymmetry [1]. The "correlation hole" that characterizes the spectral autocorrelation in the bound molecule diminishes as the average resonance width increases.

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Quantum Sublattice Systems

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Abstract

Disordered fermion systems with intrinsic sublattice symmetries are considered. We discuss the connection to conventional disordered systems, and the realization of sublattice structures in various physical contexts. We review the phenomenology of quantum interference in low-dimensional ($d \leq 2$) sublattice systems and discuss some recent theoretical approaches to the problem. In particular it is shown that the effective low energy field theory of sublattice systems sheds some light on connections between disordered electron physics, low energy QCD, and the physics of stochastic non-Hermitian operators.

The Present Status of Studies of QCD at Non-zero Densities

Ian Barbour

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Abstract

The current status of studies of QCD at non zero chemical potential is briefly reviewed. The problems associated with these studies are highlighted and discussed. Possible ways forward are reviewed

Semiclassical Approach to Dissipative Quantum Chaos

Daniel Braun

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Abstract

Dissipative chaotic quantum maps give rise to spectral statistics governed by Ginibre's ensemble of general non–hermitian and non–unitary matrices. We show how important properties of such maps can be obtained via a semiclassical theory. The analysis is based on a semiclassical approximation of the propagator of the reduced density matrix. We show that to leading order asymptotic expansion in $\hbar$ the propagator of the Wigner function is the Frobenius–Perron propagator of phase space density for the corresponding classical map. This statement holds if the Wigner functions propagated are sufficiently smooth. Several consequences arise: Traces of the quantum mechanical propagator agree with traces of the Frobenius–Perron propagator and can therefore be calculated within classical periodic theory. The Wigner transform of the invariant density matrix is a smeared out version of the classical strange attractor. And time dependent expectation values and correlation functions of observables can be evaluated by hybrid quantum–classical formulae in which the quantum character only enters via the initial Wigner function. If a classical phase–space distribution is chosen for the latter or if the map is iterated sufficiently many times the formulae become entirely classical, and powerfull classical trace formulae apply.

Irregular Scattering at Atoms and Designer Atoms

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Abstract

The investigation of irregular and chaotic dynamics in unbound systems has become the focal point of interest within the field of 'quantum chaos' or, more precisely, the quantum mechanics of classically chaotic systems. Chaotic motion in the continuum differs from bound motion in fundamental ways: the energy hypersurface is non- compact and the chaotic motion is intrinsically transient connecting the phase flows of asymtotically separable Hamiltonians in the entrance and exit channels. Accordingly, also the quantum description of irregular scattering requires features characteristically different from bound systems: While the global S matrix is unitary, its representation within the subspace of the transient " compound " system connecting to the open channels becomes non-unitary. One consequence is that the parametric dynamics of eigenvalues takes place in the complex energy plane. Signatures of transient chaos can be clearly identified in properties of the S matrix. In particular, rapidly fluctuating cross- sections can be directly related to the chaotic classical phase flow. We will present examples for Rydberg atoms interacting with a sequence of ultrashort pulses and for electron scattering at ballistic semiconductor microstructures (" designer atoms").

Eigenvector statistics in non-Hermitian random matrix ensembles

J. Chalker

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Abstract

We study statistical properties of the eigenvectors of non-Hermitian random matrices, concentrating on Ginibre's complex Gaussian ensemble, in which the real and imaginary parts of each element of an $N \times N$ matrix, J, are independent random variables. Calculating ensemble averages based on the quantity $\langle L_\alpha | L_\beta \rangle \langle R_\beta | R_\alpha \rangle$, where $\langle L_\alpha |$ and $| R_\beta \rangle$ are left and right eigenvectors of J, we show for large N that eigenvectors associated with a pair of eigenvalues are highly correlated if the two eigenvalues lie close in the complex plane. We examine consequences of these correlations that are likely to be important in physical applications.

Strongly interacting 2D Bose-luquid and Non-Hermitean Matrix Dynamic

M.V.Feigel'man and M.A.Skvortsov

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Abstract

A model Hamiltonian for 2D Bose-liquid with long-range interaction is considered, which leads to the Jastrow-type ground state eigenfunction as an exact solution. This Hamiltonian is shown to possess nonrelativistic supersymmetry and to be equivalent to the Hamiltonian that governs Dyson-type dynamics of complex eignevalues for the Gaussian ensemble of nonhermitean normal random matrices.

Systematic approach to statistics of chaotic resonances

Yan Fyodorov

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Abstract

It is shown how to solve the problem of resonance statistics in systems with broken time-reversal invariance by deriving the joint probability density of all resonances in the framework of a random matrix approach and calculating explicitly all n-point correlation functions in the complex plane. In particular, we establish the Ginibre-like statistics of resonances for many open channels. Our method is a combination of Itzykson-Zuber integration and a variant of nonlinear $\sigma-$model and can be applied when the use of orthogonal polynomials is problematic.

References: Y.V. Fyodorov and B.A. Khoruzhenko Phys.Rev.Lett. v. 83 (1999) 65

The asymptotic behaviour of the spectrum and of eigenfunctions of the Anderson-Hatano-Nelson model

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Abstract

We study the eigenvalue problem on finite intervals with periodic boundary conditions for non-Hermitian random Schrödinger operators with imaginary vector potentials in dimension one. We prove that the eigenvalues of the finite system are attracted by certain curves in the complex plane. An equation for these curves is derived and the density of non-real eigenvalues is obtained in terms of spectral characteristics of a reference Hermitian disordered system. This happens despite the fact that the spectrum of the limit operator in $l^2(\mathbf{Z})$ is a two dimensional set in the complex plane and thus is not approximated by the spectra in finite intervals. We also discuss the behaviour of the eigenfunctions of our system and the localization-delocalization transition phenomena

Imaginary vector potential and its physical applications

Naomichi Hatano

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Abstract

I introduce a non-Hermitian generalization of quantum mechanics based on an imaginary vector potential. The one-particle version of the Hamiltonian is given by

$$\mathcal{H} = \frac{1}{2m}\left(\vec{p} + \vec{A}\right)^2 + V(\vec{x}), \tag{1}$$

where the vector potential $\vec{A}$ can be a complex field. The imaginary part of $\vec{A}$ makes the Hamiltonian non-Hermite.

I show that the generalization is useful in studying various physical (Hermite) models. For example, when either $V(\vec{x})$ or the real part of $\vec{A}$ is a random potential, the model (1) is a generalization of the one-particle Anderson model of localization [1,2]. We can calculate the localization length of the *Hermitian* Anderson model by tuning the imaginary part of the vector potential. In another application, where $V(\vec{x})$ is a scattering potential for a particle, a centripetal imaginary vector potential enables us to calculate the life-time of resonance states [3].

References

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From Wigner-Dyson to Ginibre eigenvalue statistics : Gaussian case

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Abstract

By using the method of orthogonal polynomials we analyze the statistical properties of complex eigenvalues of Gaussian random matrices describing a crossover from Hermitian matrices characterized by the Wigner-Dyson statistics of real eigenvalues to strongly non-Hermitian ones whose complex eigenvalues were studied by Ginibre. Two-point statistical measures (as e.g. spectral form factor, number variance and small distance behavior of the nearest neighbor distance distribution $p(s)$) are studied in more detail.

References: Y.V. Fyodorov, B.Khoruzhenko and H.-J. Sommers Phys.Rev. Lett. v 79 (1997), 557 Ann. Inst. Henri Poincare (Phys. Theor.) v.66 (1998), 449

Universality of lifetime statistics in quantum chaotic scattering

M. Glück, A. R. Kolovsky, H. J. Korsch

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Abstract

We study a 1D model of a crystal electron in combined dc and ac fields. After an appropriate rescaling the Hamiltonian of the system can be presented in dimensionless form

$$\widehat{H} = \hat{p}^2/2 + \cos x + Fx + F_\omega x \cos(\omega t) \ , \tag{2}$$

where the momentum operator contains the scaled Planck constant $\hbar'$ as a system parameter. First we show that in the classical limit $\hbar' = 0$ the system (1) is a typical example of chaotic scattering with exponential decay. The quantum analysis of the problem is essentially more subtle and involves the condition of commensurability between the Bloch period $T_B = \hbar'/F$ and the driving period $T_\omega = 2\pi/\omega$:

$$T_\omega/T_B = r/q \ . \tag{3}$$

Provided the condition (2) is satisfied, we introduce an efficient scattering matrix of the size $q \times q$, the complex poles of which are the system quasienergy spectrum [1]. We calculate numerically the quasienergy resonances in the deep quantum ($\hbar' > 1$) and the semiclassical ($\hbar' \ll 1$) regions. For $\hbar' \ll 1$ the poles statistics and the statistics of the delay time is given for different number of channels and different system symmetries. These results are compered with the prediction of Random Matrix Theory adopted to the case of circular ensemble. For $\hbar' > 1$ the results are compared against the experimental data of Raizen's group [2], where the system (1) has been studied by using cold atoms in an accelerated optical lattice. The prospects of the laboratory study of this system in the semiclassical region of the parameters (where RMT is supposed to work) is briefly discussed.

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Level dynamics for complex matrices

Marek Kus

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Abstract

We establish level dynamics for finite matrices, employing a unified treatment of real symmetric, complex Hermitian, quaternion real, unitary, dynamics take the fictitious many-particle of the latter leads to the random-matrix theory. Ginibre's ensemble, in associated with level dynamics of arbitrary complex

Spectral Analysis of Time Correlation Functions by the Filter Diagonalization Method

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Abstract

The Filter Diagonalization Method (FDM) for spectral analysis of time signals, first introduced by Neuhauser, has extensively been developed and applied by several groups to various problems in physics and chemistry. I will review some interesting developments and applications of FDM in the area of quantum dynamics calculations and semiclassical quantization.

FDM is designed to solve the Harmonic Inversion Problem which corresponds to fitting a (time) signal by a sum of damped sinusoids. Formally being nonlinear it has though a unique solution that can be obtained by FDM. In the most efficient version FDM allows to extract the maximum information contained in the time signal of question. To optimally utilize this property one wants to produce signals (e.g., quantum time correlation function) with minimum amount of noise. For a quantum scattering problem this is complicated by the need to implement an optical potential to localize the propagated wave function, as all existing numerical schemes for the time propagators with optical potentials are approximate and therefore always introduce some noise. Interestingly, Chebyshev correlation function is conceptually similar to the time correlation function, but has the advantage of being both very accurate (i.e., zero noise) and easy to evaluate numerically. Moreover, it can be modified with absorbing boundary conditions which correspond to introducing an effective complex optical potential. The resulting Chebyshev correlation function can be shown to be exactly a sum of damped sinusoids which is ideal for FDM. this method allows to compute very accurately hundreds of resonances with moderate numerical effort.

Exactly the same method of harmonic inversion can be used in the very straight forward way to invert semiclassical signals, for instance, a Gutzwiller periodic orbit sum to yield semiclassical both resonance and bound state energies.

Transfer matrices and non Hermitian Hamiltonians: some spectral identities

L. Molinari

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Abstract

I consider general Hamiltonian matrices with tridiagonal structure made of blocks:

$$(Hu)_n = h_n u_n + l_n u_{n+1} + l^*_{n-1} u_{n-1} \tag{4}$$

where h_n, l_n are square matrices of size M, $h_n = h^*_n$, and $\det l_n$ is nonzero. I obtain a "duality relation" between the spectrum of the above Hamiltonian $H(z)$ with boundary conditions

$$u_0 = 1/z u_N, \quad u_{N+1} = z u_1 \tag{5}$$

and the spectrum of the Transfer matrix at energy E, $T(E)$:

$$\det[T(E) - z] = cost(-z)^M det[E - H(z)]. \tag{6}$$

Among several interesting consequences, I derive a generalization of Thouless relation to $M > 1$, discuss the band structure of periodic case $H(e^i\phi)$, and the channel structure for the scattering problem. The duality relation may be extended to T^*T, and offers the possibility of studying the average Lyapunov spectrum by averaging over the Hamiltonian ensemble.

Interfering resonances in a quantum billiard

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Abstract

We present a method for numerically obtaining the positions, widths and wavefunctions of resonance states in a two dimensional billiard connected to a waveguide. For a rectangular billiard, we study the dynamics of resonance poles as a function of increasing coupling strength between the waveguide and the billiard.

Non-Hermiticity and Disorder

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Abstract

The exceptional properties of double poles of the S-matrix are known for a long time but their influence on the spectra of quantum systems at high level density is studied only recently. A hint to them is the avoided level crossing appearing for states below as well as above particle decay thresholds at a critical value $\alpha_{\rm cr}$ of a certain parameter α [1]. It reflects the strong mutual influence of one state to another one and can be traced directly in the spectra of a microwave resonator [2] and also in the laser induced manipulation of spectra of atoms [3].

If the avoided level crossing is caused by the direct interaction of the two states then level repulsion along the real energy axis occurs for $\alpha > \alpha_{\rm cr}$. This fact is very well known and basic for the discussion of quantum chaos in closed quantum systems with many states. If the two levels interact, however, mainly via the continuum, level repulsion along the imaginary energy axis appears. In this case, different time scales are formed at $\alpha > \alpha_{\rm cr}$. One of the states will be trapped while the other one aligns with one decay channel and becomes short-lived. In a system with many states, K states align eventually with the K open decay channels and the remaining ones become trapped.

In atoms in a strong laser field, both the direct interaction of the states as well as the interaction via the continuum may be important. Additional interference phenomena will therefore occur which lead, under certain conditions, to population trapping [3]. In this case, the width of one of the states vanishes at a finite value $\alpha > \alpha_{\rm cr}$. Thus, population trapping is a signature of a double pole of the S matrix *and* can be studied experimentally.

If the crossing of the states is avoided not only for two states but for all states of the system at (almost) the same value $\alpha_{\rm cr}$ (i.e. the S matrix has a multiple pole instead of a double pole), then the quantum system suffers a second-order phase transition [4]. This may happen in the one-channel case. The characteristic features of the phase transition are well expressed already for a finite number of states (about 100) as numerical studies showed [1,4].

* Supported by DFG and SMWK

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Field Theory of Non-Hermitian Operators

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Abstract

The spectral and localisation properties of non-Hermitian operators differ substantially from their Hermitian counterparts. These unusual properties can be attributed to the existence of an implicit chiral symmetry exhibited by all non-Hermitian operators. We discuss a supersymmetrical field theoretic formulation of non-Hermitian structures which reveals a general classification scheme and exposes fundamental connections to a variety of Hermitian model systems including sublattice models and superconductors. Applications of this approach to the spectral properties of simple models of random classical dynamics are discussed.

A Simple Mode on a Background of Unstable chaotic state

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Abstract

Simple states of predominantly 1p-1h structure embedded into continuum are typical for mesoscopic many-fermion quantum systems. When studied experimentally with the aid of various reactions, such a simple exitation is associated with a specific signal and reveal enhanced cross sections in specific channels at corresponding resonance energy.

Due to the coupling to complicated compound states in the same energy range, the simple mode, being fragmented over many exact stationary states, acquires an internal damping. As a result, the original state dissolves in the background and the exitation function becomes related to the wide average strength distribution of the original "label" smoothly depending on exitation energy.

However, the situation described changes appreciably in the realistic case with background states decaying through many evaporation channels. We consider here the most general case when both simple and background states are coupled to the continuum and have finite lifetimes. The interplay between internal and external dynamics is investigated using a general formalism of the effective non-Hermitian Hamiltonian and looking at the situation both from "inside" (strength functions) and from "outside" (cross sections and delay times). The widths of the intrinsic background states qualitatively modify the strength function of the simple mode restoring its individuality.

One has to project the dynamics of quasistationary intrinsic states with complex eigenenergies back to the real energy axis for determining the observables, such as cross sections and delay times, measured in a scattering experiment. In such a way one tracks the intrinsic dynamics in manifestations of the simple mode in various situations. The restoration of isospin purity and disappearance of the collective strength of giant resonances at high excitation energy are important particular examples.

Density of S-matrix poles in quantum chaotic scattering

Hans-Jürgen Sommers und Yan Fyodorov

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Abstract

Assuming the validity of random matrix theory for describing the statistics of a *closed* chaotic quantum system, we study analytically the statistics of S-matrix poles characterizing scattering in its *open* counterpart. By using the supersymmetry method we derive an explicit expression for the density of S-matrix poles (resonances) in the complex energy plane. When all scattering channels are considered to be equivalent our expression describes a crossover from the χ^2 distribution of resonance widths (regime of isolated resonances) to a broad power-like distribution typical for the regime of overlapping resonances. The first moment is found to reproduce exactly the Moldauer-Simonius relation between the mean resonance width and the transmission coefficient. We consider the cases of broken and conserved time reversal invariance. In the latter case a new supersymmetric parametrisation has to be introduced.

Random Matrices and the Phase Diagram of QCD

M. Stephanov

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Abstract

We show how a certain class of random matrix models can be used to describe the dynamics of the chiral symmetry breaking and restoration in QCD at finite temperature and baryon density. An explicit solution of a corresponding non-hermitian random matrix model provides a lucid explanation for a long-standing puzzle in lattice QCD simulations at finite baryon density.

Random Matrix Theory, Determinants and Chiral symmetry

Jac Verbaarschot

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Abstract

I will give an overview of the application of Random Matrix Theory to QCD both at zero and nonzero chemical potential and discuss its connections with a low-energy effective theory dictated by chiral symmetry. We will mainly focus on QCD Dirac spectra. Universality will be related to the existence of a mass gap. Deviations from Random Matrix Theory will be discussed. One way QCD differs from disordered condensed matter systems is by the presence of a fermion determinant in the partition function. Its importance for the low-energy behavior of the partition function will be discussed. This is most dramatically illustrated by non-Hermitean random matrix theories.

Dynamical stabilization of the kicked hydrogen atom

S.Yoshida, C. O. Reinhold, J. Burgdoerfer

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Abstract

We analyze the quantum dynamics of the one-dimensional periodically kicked Rydberg atom. The time-dependent Schrodinger equation is solved by employing a non-unitary representation of the period-one evolution operator in a finite basis set which accounts for the outbound probability flux. We find a direct correspondence between stable classical islands in phase space and the quantum Husimi distributions of stable Floquet states. More surprisingly, we find quantum localization for very high- frequency perturbation. In this case, stable Floquet states are localized in the region of phase space where the corresponding classical Poincare map displays a chaotic sea. The relation between quantum localization and the unstable periodic points will be discussed.

On the Structure of Correlation Functions in the Normal Matrix Model

O. Zaboronsky

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Abstract

The structure of Normal Matrix Model is discussed. The universality of the correlation functions in the large N scaling limit is proven. New multi-dimensional completely integrable systems connected to the model are constructed. The description of the space of solution is presented.

Disorder, Symmetries and Nonlinear Sigma Models

M.R. Zirnbauer

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Abstract

This talk is based on the author's article J. Math. Phys. 37 (1996) 4986. It is argued that each of the ten known universality classes for noninteracting disordered quasiparticle systems maps on a field theory of the nonlinear sigma model type, where the fields take values in one of the ten large families of Riemannian symmetric superspaces. This general classification scheme includes non-Hermitian systems.