Transfer matrices and non Hermitian Hamiltonians: some spectral identities

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Abstract

I consider general Hamiltonian matrices with tridiagonal structure made of blocks:

$$(Hu)_n = h_n u_n + l_n u_{n+1} + l^*_{n-1} u_{n-1} \tag{1}$$

where h_n, l_n are square matrices of size M, $h_n = h^*_n$, and $\det l_n$ is nonzero. I obtain a "duality relation" between the spectrum of the above Hamiltonian $H(z)$ with boundary conditions

$$u_0 = 1/z u_N, \quad u_{N+1} = z u_1 \tag{2}$$

and the spectrum of the Transfer matrix at energy E, $T(E)$:

$$\det[T(E) - z] = cost(-z)^M det[E - H(z)]. \tag{3}$$

Among several interesting consequences, I derive a generalization of Thouless relation to $M > 1$, discuss the band structure of periodic case $H(e^i\phi)$, and the channel structure for the scattering problem. The duality relation may be extended to T^*T, and offers the possibility of studying the average Lyapunov spectrum by averaging over the Hamiltonian ensemble.