I Special Relativity

1.1 Recall Maxwell equations (1873):

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon} \quad \text{Gauss (electric)} \quad (1.1) \]

\[ \nabla \cdot \vec{B} = 0 \quad \text{Gauss (magnetic)} \quad (1.2) \]

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{Faraday} \quad (1.3) \]

\[ \nabla \times \vec{B} = \mu \left( \frac{\partial \vec{E}}{\partial t} + \varepsilon \frac{\partial \vec{E}}{\partial x} \right) \quad \text{Ampère} \quad (1.4) \]

Here, \( \nabla \times \vec{E} \) is the curl of \( \vec{E} \), \( \nabla \cdot \vec{E} \) is the gradient of \( \vec{E} \).

Maxwell equations can be combined to

(see courses: PHY301, MAT 203/205)

\[ \Delta \vec{E} = \varepsilon \mu \frac{\partial^2}{\partial x^2} \vec{E} = 0 \quad (1.5) \]

This is a wave equation with solution

\[ \vec{E}(x,t) = \vec{E} = \vec{E}_0 \sin (x-ct + \text{const.}) \quad (1.6) \]

The velocity of this electromagnetic wave is

\[ c = \frac{1}{\sqrt{\varepsilon \mu}} \]

\[ = 2.998 \cdot 10^8 \frac{m}{s} \quad \text{in free space} \quad (1.8) \]

Known to Maxwell, since permeability \( \mu \) and permittivity \( \varepsilon_0 \) could be estimated.
Fizeau (1850): measures velocity of light

which coincides with \( c \approx 3 \times 10^8 \text{ m/s} \)

Hertz (1888): • created electromagnetic waves with oscillating electric currents
• showed that electromagnetic waves are
  → reflected by metal mirrors
  → refracted by dielectrics like glass
  → exhibit polarization and interference
  just like light.

⇒ 19th century knowledge: light is an electromagnetic wave

19th century experience with waves:
(such as sound waves, waves on water, ...)
all waves are carried by a medium.

But: Hertz had shown that electromagnetic waves propagate through free empty space.

Q: Which medium carries light waves?

⇒ Ether Hypothesis
I.2. Michelson-Morley Experiment

To test ether hypothesis, measure relative velocity \( v \) of earth w.r.t. ether.

Naive estimate of \( v \): \( v \approx 3 \cdot 10^4 \text{ m/s} \)  
velocity of earth around the sun

Interference experiment.

\[ \text{coherent light source} \]

\[ \text{Arm 1} \]

\[ \text{Mirror 1} \]

\[ \text{Arm 2} \]

\[ \text{Mirror 2} \]

\[ \text{Detector} \]

Assume that both arms have same length \( L \) and that earth is moving parallel to arm 2.

How long does the light travel?

\[ (1.9) \quad \text{Arm 2: } t_2 = \frac{L}{c+v} + \frac{L}{c-v} = \frac{2Lc}{c^2-v^2} = \frac{2L}{c} \left( \frac{1}{1-\frac{v^2}{c^2}} \right) \]

\[ (1.10) \quad \text{Arm 1: } t_1 = \frac{\sqrt{L^2-v^2}}{c} = \frac{2L}{c} \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \]
Time difference between Arm 1 and Arm 2:

\[(1.11) \quad \Delta t = |x_2 - x_1| \approx \frac{4\pi L}{c^2} \quad \text{(Rec)}\]

Path difference:

\[(1.12) \quad \Delta d = c \Delta t = \frac{4\pi L}{c^2}\]

For their experiment \((L = 11\text{ m})\), Michelson and Morley expected \(\Delta d \approx 220\text{nm} \approx \frac{1}{3}\) of wavelength of visible light \(\Rightarrow \) significant interference between light from Arm 1 and 2 \(\text{(fringe shift \(\approx 0.3-0.4\))}\)

Experimental accuracy more than factor 10 better?

But: Michelson and Morley never measured any shift.

Einstein's conclusion (1905):

"... the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest."