The change in kinetic energy equals the work done by the force \( F = \frac{dp}{dt} \)

\[
W = \int_{x_1}^{x_2} \frac{dp}{dx} \, dx
\]

use: \( u = \frac{dx}{dt} \) and \( p = m_0 u \)

\[
= \int_0^{v_f} dp \cdot u
\]

use: \( dp = \frac{m_0}{(1-\frac{u^2}{c^2})^{\frac{3}{2}}} \, du \) see (1.50)

\[
= m_0 \int_0^{v_f} \frac{u \, du}{(1-\frac{u^2}{c^2})^{\frac{3}{2}}}
\]

\[
= \frac{m_0 c^2 \left[ \frac{1}{\sqrt{1-\frac{u_f^2}{c^2}}} \right]_{u=u_f}}{\sqrt{1-\frac{v_f^2}{c^2}}} - m_0 c^2
\]

\[
= \frac{m_0 c^2}{\sqrt{1-\frac{v_f^2}{c^2}}} - m_0 c^2
\]

(1.51)

So, the kinetic energy is the total minus the rest energy, i.e. the rest energy can be viewed as potential energy.

Expand (1.51) for \( v_f \ll c \)

\[
K = W \approx m_0 c^2 \left( 1 + \frac{1}{2} \frac{v_f^2}{c^2} + O\left( \frac{v_f^4}{c^4} \right) \right) - m_0 c^2
\]

(1.52)

\[
\approx \frac{1}{2} m_0 v_f^2
\]

In the non-relativistic limit, we regain the Newtonian expression for the kinetic energy.
iv) Applications of relativistic energy-momentum conservation:

a) Fission

Consider nucleus of mass \(M\), decaying (i.e. undergoing fission) into particles of mass \(M_1, M_2, M_3\) and velocities \(u_1, u_2, u_3\). Conservation of total energy implies [see (0.38)]

\[
M c^2 = \frac{M_1 c^2}{\sqrt{1 - \frac{u_1^2}{c^2}}} + \frac{M_2 c^2}{\sqrt{1 - \frac{u_2^2}{c^2}}} + \frac{M_3 c^2}{\sqrt{1 - \frac{u_3^2}{c^2}}}
\]

(1.53)

This implies \(M - (M_1 + M_2 + M_3) \geq 0\). In fission reactions, mass is 'lost' and transferred into energy of motion.

The disintegration energy released per fission is denoted by the \(Q\)-value:

\[
Q = \left[ M - (M_1 + M_2 + M_3) \right] c^2 = \Delta m \cdot c^2
\]

(1.54)

Example: \(^{235}_{92} \text{U} \rightarrow ^{90}_{37} \text{Rb} + ^{143}_{55} \text{Cs} + 3^1_n\)

Masses are given in terms of the atomic mass unit

\[
u = 1.660 \times 10^{-27} \text{kg} = 931.5 \text{MeV}/c^2
\]

\[
\Delta m = M_u - (M_{\text{Rb}} + M_{\text{Cs}} + 3m_n) \quad \text{these masses are known}
\]

\[
= 236.045 \nu - (89.915 + 142.927 + 3 \times 1.009) \nu
\]

\[
= 1.775 \nu = 2.947 \times 10^{-28} \text{kg} = 165.4 \text{MeV}/c^2
\]

\(Q = \Delta m \cdot c^2 = 165.4 \text{MeV}\)
To visualize \( Q = 165.4 \text{ MeV} \), consider 1 kg. of \(^{235}\text{U}\). There are 236 g uranium per mol and \(6.022 \times 10^{23}\) nucleons per mol:

\[
N = \frac{6.022 \times 10^{23} \text{ nucleons}}{236 \text{ g/mol}} \cdot (1000 \text{ g}) = 2.55 \times 10^{24} \text{ nucleons}
\]

Energy \( = N \cdot Q = 2.55 \times 10^{24} \cdot 165 \text{ MeV} \cdot \frac{4.45 \times 10^{18} \text{ kWh}}{\text{MeV}} \approx 1.8 \times 10^7 \text{ kWh} \)

b.) Binding energy (BE)

The mass of any nucleus is less than the sum of its component neutrons and protons by a difference \( \Delta m \). An energy \( \Delta m c^2 = BE \) must be supplied to the nucleus in order to dissociate the nucleus into its components:

\[
MC^2 + BE = \sum_{i=1}^{n} m_i c^2
\]

- Eq. (1.56) is valid for any bound state irrespective of the forces which keep it together.

- If repulsion between particles can be overcome, (1.56) allows us to liberate huge quantities of energy \( \Rightarrow \text{ fusion} \)

\[
^1\text{H} + ^1\text{H} \rightarrow ^2\text{He} + 23.9 \text{ MeV}
\]