Problems:

1. Exercise with units: (a) Show that the expressions \((\frac{\hbar}{c})^{1/2}\), \((\frac{\hbar}{G})^{1/2}\) and \((\frac{\hbar c}{G})^{1/2}\) have dimensions of length, time and mass \((G\) is the gravitational constant). Determine their numerical value and make a wild guess of what could be their physical meaning.

(b) Calculate the energy of a photon whose wavelength is 500 nm. Express your answer in units of electron volts and in units of Joule. What is the frequency of this photon?

2. Following the logic of section II.6 of the lecture, calculate \(n(E) dE\), the average number density of photons with energy between \(E\) and \(E + dE\) in a cavity at temperature \(T\). [Hint: you have to multiply the average number density per mode by the number of modes per phase space.] Determine from this expression the number density \(n\) of photons of arbitrary energy. What is the temperature dependence of \(n\). Calculate the average number of photons per cm\(^3\) for a cavity of temperature \(T = 0.1\) K, \(T = 1\) K and room temperature \(T = 300\) K. Use \(\int_0^\infty \frac{2}{e^{z/2}-1} \approx 2.4\).

3. A light bulb has an output of 100 W. Its energy distribution peaks at a wavelength \(\lambda = 1000\) nm. (a) What is the temperature of the filament? (b) What is the surface of the filament? (c) How many photons does the light bulb emit per second?

4. How much energy is needed to heat up 1 mol of a solid from \(T = 100\) K to \(T = 103\) K? Answer this question based (a) on the classical theory of heat of Dulong-Petit (b) Einstein’s quantum theory of heat for a solid of Einstein temperature \(\Theta_E = 300\) K. Hint: in case that you can’t do the integral of the problem (b) exactly, use \(T/\Theta_E \ll 1\).

5. The work function is defined as the energy with which electrons are bound in a solid. Consider the metals lithium, beryllium, and mercury, which have work functions of 2.3 eV, 3.9 eV and 4.5 eV, respectively. (a) What is the cut-off frequency below which the photoelectric effect is not observed?

If light of wavelength 300 nm is incident on each of these metals, determine (b) which metals exhibit the photoelectric effect? (c) the stopping voltage for the different materials? (d) the maximum kinetic energy for the photoelectron in each case.

6. An excited iron (Fe) nucleus (mass 57 u) decays to its ground state with the emission of a photon. The energy available for this transition is 14.4 keV. (a) By how much is the photon energy reduced from the full 14.4 keV as a result of having to share energy with the recoiling atom? (b) What is the wavelength of the emitted photon?