1. Tunneling through barriers which are high or wide (or both) is very unlikely.
   (a) Starting from eq. (4.56) of the lecture notes for the transmission coefficient, show
   that for a square barrier with $2mUL^2/h^2 \gg 1$ and particle energies well below the top of
   the barrier ($E \ll U$), the probability of transmission is approximately
   \[ P = 16 \frac{E}{U} \exp \left( -2 \sqrt{\frac{2m(U - 1)}{h}L} \right). \]
   (b) Give numerical estimates for the exponential factor in $P$ for each of the following
   cases: 1.) an electron with $U - E = 0.01$ eV and $L = 0.1$ nm; 2.) an electron with
   $U - E = 1$ eV and $L = 0.1$ nm; 3.) an α-particle ($m = 6.7 \times 10^{-27}$ kg) with $U - E = 10^6$
   eV and $L = 10^{-15}$ m; 4.) a bowling ball ($m = 8$ kg) with $U - E = 1$ J and $L = 2$ cm
   (this corresponds to the ball’s getting past a barrier of 2 cm wide and too high for the
   ball to slide over).

2. The linear operator corresponding to angular momentum is \( \hat{\mathbf{L}} = -i \hbar \hat{\mathbf{r}} \times \hat{\nabla} = (\hat{L}_x, \hat{L}_y, \hat{L}_z) \).
   Derive the commutation relations:
   \[
   [\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z,
   [\hat{L}_z, \hat{L}_x] = 0.
   \]

3. Consider an infinitely extended potential step of the form $V(x) = 0$ for $x < 0$ and
   $V(x) = V$ for $x > 0$. A particle of mass $m$ and energy $E$ is incident from the left onto
   this potential step. Determine the reflection coefficient for the case $E > V$. To this
   end, specify solutions of time-independent Schrödinger equation, determine continuity
   conditions and solve them.

4. Consider a particle incident from the left on a square barrier of width $L$ and height $U$.
   The particle has energy $E > U$. Specify the time-independent Schrödinger equation and
   determine the wave function which solves it by specifying the continuity conditions at
   $x = 0$ and $x = L$. Calculate the transmission coefficient and show that transmission is
   perfect ($T = 1$) for specific energies. What is different for the case $E > U$ considered
   here, compared to the case $E < U$ considered in the lecture?

5. Recall Heisenberg’s uncertainty principle $\Delta p_x \Delta x \geq \frac{\hbar}{2}$. An air rifle is used to shoot 1.0
   g particles at 100 m/s through a hole of diameter 2.0 mm. How far from the rifle must
   an observer be to see the beam spread by 1.0 cm because of the uncertainty principle?
   Compare this answer with the diameter of the Universe ($\approx 10^{26}$ m).

6. Don’t forget to practice energy-momentum conservation: A particle of mass $m$ moving
   along the $x$-axis with a velocity component $+u$ collides head-on and sticks to a particle
   of mass $m/3$ moving along the $x$ axis with the velocity component $-u$. What is the mass
   $M$ of the resulting particle?