Solutions Homework 10

1. $$\Delta t = \frac{\Delta t_{clock}}{\sqrt{1-\frac{v^2}{c^2}}} = \Delta t_{clock} + \frac{1}{2}\frac{v^2}{c^2}\Delta t_{clock}$$

The observer on ground measures a time interval Δt larger by

$$\frac{1}{2}\frac{v^2}{c^2}\Delta t_{clock} = \frac{1}{2}\frac{(400\,\frac{m}{s})^2}{(3\cdot 10^8\,\frac{m}{s})^2}\,3600\,s$$

$$= \frac{1}{2}\,\frac{16}{9}(10^{-6})^2\cdot 36\cdot 10^2\,s$$

$$= 3.2\cdot 10^{-9}\,s = 3.2\,ns$$

2.) Lorentz trafo
$$x' = \gamma(x - vt)$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

a.) $x_1' = 0 \qquad x_2' = \gamma\cdot 100\,m = \frac{1}{\sqrt{1-0.49}}\,100\,m$ #

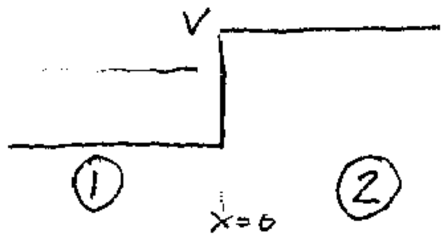
b.) events are separated by
$$x_2' - x_1' = \frac{1}{\sqrt{1-0.49}}\,100\,m = 140\,m$$

c.) $t_1' = 0 \qquad t_2' = -\gamma\,\frac{vx_2}{c^2}$

Event 2 is earlier by

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{vx_2}{c^2} = \frac{1}{\sqrt{1-0.49}}\cdot\frac{0.7}{3\cdot 10^8\,\frac{m}{s}}\cdot 100\,m = 0.32\cdot 10^{-6}\,s$$

$$= 0.32\,\mu s$$

3.) 

$$-\frac{\hbar^2}{2m}\Delta\Psi(x) + V(x)\Psi(x) = E\Psi(x)$$

in ①: $-\frac{\hbar^2}{2m}\Delta\Psi(x) = E\Psi(x)$

$$\Rightarrow \Psi_I(x) = Ae^{ikx} + Be^{-ikx} \qquad k = \frac{\sqrt{2mE}}{\hbar}$$

in ②: $-\frac{\hbar^2}{2m}\Delta\Psi(x) = -(V-E)\Psi(x)$, where $(V-E)$ positive

$$\Rightarrow \Psi_{II}(x) = Ce^{-\alpha x} + De^{\alpha x} \qquad \alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$$

$D=0$ for physics reasons (probability cannot increase exponentially for $x \to \infty$)

continuity conditions:

$$\Psi_I(0) = \Psi_{II}(0) \quad \Rightarrow \quad A+B = C$$

$$\frac{\partial\Psi_I}{\partial x}(0) = \frac{\partial\Psi_{II}}{\partial x}(0) \quad \Rightarrow \quad ik(A-B) = \alpha C$$

Combine: $\alpha(A+B) = ik(A-B) \quad \Rightarrow A(\alpha-ik) = B(-ik-\alpha)$

$$R = \frac{BB^*}{AA^*} = \frac{\alpha-ik}{-ik-\alpha}\cdot\frac{(\alpha-ik)^*}{(-ik-\alpha)^*} = 1$$

$$|\Psi_{II}^*(x)\Psi_{II}(x)| \propto \exp\left[-2\underbrace{\frac{\sqrt{2m(V-E)}}{\hbar}}_{=1/D}x\right]$$

penetration depth D

4.) $E_{rot-vib} = \frac{\hbar^2}{2I_{cm}} \ell(\ell+1) + (v+\frac{1}{2})\hbar\omega$

a.) $\Delta E_{rot-vib}^{\ell=6\to 5} = \frac{\hbar^2}{2I_{cm}}\left(6\cdot 7 - 5\cdot 6\right) = hf = h\frac{c}{\lambda}$, $\lambda = 1.35\,cm$

$= \frac{\hbar^2}{2I_{cm}} \cdot 12$

$\Delta E_{rot-vib}^{\ell=0\to 1} = \frac{\hbar^2}{2I_{cm}} \cdot 2 = h\frac{c}{\lambda'}$

$\Rightarrow \lambda' = 6\cdot\lambda = 8.10\,cm$ $\qquad f' = \frac{c}{\lambda'} = 3.70\,GHz$

$= \frac{3\cdot 10^{10}\frac{cm}{s}}{8.1\,cm} = 3.7\cdot 10^{9}\frac{1}{s}$

b.) $I_{cm} = \frac{\hbar^2}{hf'} = \frac{\hbar}{2\pi f}$

$= \frac{1.055\cdot 10^{-35}\,Js}{2\cdot\pi\cdot 3.7\cdot 10^{9}\frac{1}{s}} = 4.5\cdot 10^{-46}\,Js^2$ $\qquad 1J = 1\frac{kg\,m^2}{s^2}$

$= 4.5\cdot 10^{-46}\,kg\,m^2$

5.) $I_{cm} = \underbrace{\frac{m_H m_{Cl}}{m_H + m_{Cl}}}_{\mu = \text{reduced mass}} \cdot R_0^2$, $R_0 = 0.1275\,nm$

$\omega = \frac{L}{I_{cm}} = \frac{\hbar\sqrt{\ell(\ell+1)}}{I_{cm}} = \frac{\sqrt{2}\,\hbar}{I_{cm}}$

$\mu = \frac{1.0079\cdot 35.453}{1.0079 + 35.453}\,u = 0.98\,u = 0.98\cdot 931.5\,\frac{MeV}{c^2}$

$$I_{cm} = 0.98 \cdot 931.5 \cdot 10^{6}\, eV \cdot \frac{1}{9 \cdot 10^{16}} \cdot \frac{s^2}{m^2} \cdot (0.1275)^2 10^{-18} m^2$$

$$= 1.64 \cdot 10^{-28} eV s^2$$

$$\omega = \frac{L}{I_{cm}} = \frac{\sqrt{2}\hbar}{I_{cm}} = \frac{\sqrt{2} \cdot 6.582 \cdot 10^{-16} eV \cdot s}{1.64 \cdot 10^{-28} eV s^2}$$

$$= \sqrt{2} \cdot 4.01 \cdot 10^{12} \frac{1}{s} = 5.67 \cdot 10^{12} \frac{1}{s}$$

6.) $E_{vib} = (v + \frac{1}{2})\hbar\omega = 4.5\, eV$

$$\hbar\omega = 6.582 \cdot 10^{-16} eV \cdot s \cdot 8.277 \cdot 10^{14} \frac{1}{s}$$

$$= 54.479 \cdot 10^{-2} eV$$

$$\Rightarrow (v + \frac{1}{2}) = \frac{4.5}{0.54479} = \underline{\underline{8.26}}$$

This shows that molecule can be excited like a harmonic oscillator (quantum levels are allowed), but that it can't be excited too high, and that anharmonic ~~an~~ features may show up for $v < 7$.