Solutions, Homework 5

1. a) Wavelength at rest: $\lambda = 394\,nm$

red-shifted: $\lambda_A = 475\,nm$, $\lambda_B = 500\,nm$

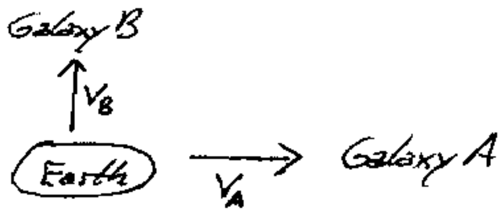
recall Doppler-effect: $f' = \sqrt{\dfrac{1+\frac{v}{c}}{1-\frac{v}{c}}}\, f \Rightarrow f'^2(1-\frac{v}{c}) = f^2(1+\frac{v}{c})$

solve for velocity v: $\dfrac{v}{c} = \dfrac{f'^2 - f^2}{f'^2+f^2} = \dfrac{\lambda^2-\lambda'^2}{\lambda^2+\lambda'^2}$

$$v_A = \frac{475^2 - 394^2}{475^2+394^2}c = 0.185\,c$$

$$v_B = \frac{500^2-394^2}{500^2+394^2}c = 0.234\,c$$

b.)



in earth system

Lorentz transformation from earth to A:

$$x_A = \gamma(x - v_A t) \qquad \gamma = \frac{1}{\sqrt{1-\frac{v_A^2}{c^2}}}$$
$$y_A = y$$
$$z_A = z$$
$$t_A = \gamma\left(t - \frac{v_A x}{c^2}\right)$$

Velocity transformation from $\vec{u} = \dfrac{d\vec{x}}{dt}$ to $\vec{u}_A = \dfrac{d\vec{x}_A}{dt}$

$$u_A^x = \frac{dx_A}{dt_A} = \frac{\gamma(dx - v_A dt)}{\gamma(dt - \frac{v_A dx}{c^2})} = \frac{u_x - v_A}{1-\frac{v_A u_x}{c^2}}$$

$$u_A^y = \frac{dy_A}{dt_A} = \frac{dy}{\gamma(dt - \frac{v_A dx}{c^2})} = \frac{u_y}{\gamma(1-\frac{v_A u_x}{c^2})}$$

Now, $\vec{u}$ is velocity of B in Earth-based system, $\vec{u} = (0, v_B)$, so in system A:

$$u_A^x = \frac{-v_A}{1 - \frac{v_A \cdot 0}{c^2}} = -v_A$$

$$u_A^y = \frac{v_B}{\gamma\left(1 - \frac{v_A \cdot 0}{c^2}\right)} = \sqrt{1 - \frac{v_A^2}{c^2}} \cdot v_B = 0.230$$

$$|u_A| = \sqrt{{u_A^x}^2 + {u_A^y}^2} = \sqrt{v_A^2 + \left(1 - \frac{v_A^2}{c^2}\right)v_B^2}$$ now put in numbers

$$= 0.295\,c$$

c.) Wavelength seen by observer on A: use $f = \frac{c}{\lambda}$

$$\lambda_{AB} = \sqrt{\frac{1 + \frac{u}{c}}{1 - \frac{u}{c}}}\,\lambda$$

$$= \sqrt{\frac{1 + 0.295}{1 - 0.295}}\ 394\,\text{nm}$$

$$= 1.355 \cdot 394\,\text{nm}$$

$$= \underline{534\,\text{nm}}$$

2. $m_1 = 900\,kg$ $\quad v_1 = 0.85c$ $\quad m_2 = 1400\,kg$

total energy: $E_t = E_1 + E_2 = \frac{m_1 c^2}{\sqrt{1-\frac{v_1^2}{c^2}}} + m_2 c^2$

total momentum: $p_t = p_1 + p_2 = \frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} + 0$

numbers: $E_t = \left(\frac{900}{\sqrt{1-0.85^2}} + 1400\right) kg\, c^2 = 3108.5\ kg\, c^2$

$$p_t = \frac{900}{\sqrt{1-0.85^2}}\ kg\, c \ast 0.85 = 1452.21\ kg\, c$$

Now: $E_t = \frac{m_t c^2}{\sqrt{1-\frac{v_t^2}{c^2}}}$ $\quad p_t = \frac{m_t v_t}{\sqrt{1-\frac{v_t^2}{c^2}}}$

To calculate speed of composite object

$$v_t = \frac{p_t}{E_t} \cdot c^2 = \frac{1452.2}{3108.5}\, c = 0.467\, c$$

Now calculate m_t:

$$m_t = \frac{E_t}{c^2}\sqrt{1-\frac{v_t^2}{c^2}} = \frac{E_t}{c^2}\ 0.884$$

$$= 2748\ kg$$

$> m_1 + m_2$, since a lot of kinetic energy was transformed into rest energy in this gedanken experiment.

3. Photon initial energy $E_i = 0.1\,\text{MeV} = hf = h\frac{c}{\lambda}$

$$\Rightarrow \lambda = \frac{hc}{E_i} = \frac{4.136\cdot 10^{-15}\,eV\cdot s\cdot 3\cdot 10^{8}\,\frac{m}{s}}{10^{5}\,eV} = 12.4\cdot 10^{-12}\,m = 10^{-3}\cdot 12.4\,nm = 0.0124\,nm$$

Now, Compton's formula (2.60) notes: $\frac{h}{m_e c}(1-\cos\Theta) = \lambda' - \lambda$

$$\Rightarrow E_f = \frac{hc}{\lambda'} = \frac{hc}{\frac{h}{m_e c}(1-\cos\Theta)+\lambda} = \frac{1}{\frac{1}{m_e c^2}\frac{1}{2} + \frac{1}{E_i}}$$

$$= \frac{E_i\cdot m_e c^2}{\frac{1}{2}E_i + m_e c^2} = \frac{0.1\cdot 0.511}{0.05+0.511}\,\text{MeV} = \underline{\underline{0.0919\,\text{MeV}}}$$

Now, use momentum conservation in $\perp$ and $\parallel$ direction

$$\frac{E_f}{c}\sin\Theta = p_e \sin\varphi$$

$$\frac{E_i}{c} = \frac{E_f}{c}\cos\Theta + p_e\cos\varphi \qquad \sin^2\Theta = \frac{3}{4}$$

$$p_e^2 = \left(\frac{E_f}{c}\right)^2\sin^2\Theta + \left(\frac{E_i}{c} - \frac{E_f}{c}\cos\Theta\right)^2 = \left((0.099)^2\cdot\frac{3}{4} + \left(0.1-0.099\cdot\frac{1}{2}\right)^2\right)\frac{\text{MeV}^2}{c^2}$$

$$c^2 p_e^2 = 0.00910\,\text{MeV}^2$$

$$E_{kin} = \sqrt{m_e^2c^4 + p_e^2c^2} - m_e c^2$$

$$= \left(\sqrt{0.511^2 + 0.0091^2} - 0.511\right)\text{MeV} = \cancel{0.00009579} \;\; 8.91\,\text{keV}$$

$$\cancel{= 95.9\,eV}$$

$$\sin\varphi = \frac{E_f\sin\Theta}{c\cdot p_e} = \frac{0.0919\cdot\frac{\sqrt{3}}{2}}{\sqrt{0.00919}} = \cancel{0.861}\; 0.823$$

$$\text{Arc}\sin 0.861 = \varphi = \cancel{1.03858} = \cancel{0.33059}\,\pi$$

$$0.966 \Rightarrow \varphi = 55^\circ$$

4. i) momentum of hockey puck non-relativistic expression is appropriate

$$p = 0.170\,kg \cdot 160 \cdot \frac{10^3 m}{60 \cdot 60\, s} = 7.56 \cdot kg \frac{m}{s}$$

$$\lambda = \frac{h}{p} = \frac{4.136 \cdot 10^{-15} eV \cdot s}{p} \cdot \frac{1.783 \cdot 10^{-30} kg}{1\, MeV/c^2}$$

$$= \frac{4.136 \cdot 1.783}{7.56} 10^{-51} \frac{s}{kg\,m} \cdot s\,kg \cdot \frac{m^2}{s^2} \cdot c^2$$

$$= \underline{\underline{0.975 \cdot 10^{-51} m}} \quad \cdot 9 \cdot 10^{16}$$

$$= \underline{\underline{8.8 \cdot 10^{-35} m}}$$

far too small to be relevant for a NHL game or any other measurement.

ii) momentum of electron

$$p = \frac{h}{\lambda} = \frac{4.136 \cdot 10^{-15} eV \cdot s}{10^{-14} m} = .4136\, eV \cdot \frac{s}{m} = .4136 \cdot \frac{3 \cdot 10^8}{c} eV$$

$$= \frac{m_e v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

calculate v, then relat. kin. ene.

$$p^2 \left(1 - \frac{v^2}{c^2}\right) = m_e^2 v^2 \qquad p^2 = v^2 \left(m_e^2 + \frac{p^2}{c^2}\right)$$

$$v^2 = \frac{p^2}{m_e^2 + \frac{p^2}{c^2}} = \frac{(41.36 \cdot 3 \cdot 10^8)^2}{(0.511 \cdot 10^6)^2 + (41.36 \cdot 3 \cdot 10^8)^2} c^2 \sim 1\, c^2$$

$0.99998 c^2$

⇒ treat electron as mass-less object since mass negligible

$$E = p \cdot c \approx E_{kin} = 124\, MeV$$

much larger than binding energy ⇒ electron will escape!

4 iii) de Broglie wavelength $\lambda = 10^{-10}\,m$

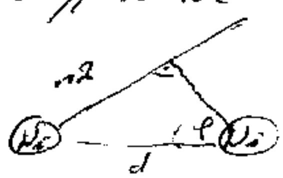
$$p = \frac{h}{\lambda} = \frac{4.136\cdot 10^{-15}\,eVs}{10^{-10}\,m} = 4.136\cdot 10^{-5}\,eV \cdot \frac{3\cdot 10^{8}}{c}$$

$$= 1.24\cdot 10^{4}\,\frac{eV}{c}$$ this is a non-relat. mom

$$E_{kin} = \frac{1}{2}\,\frac{p^2}{m_e} = \frac{1}{2}\,\frac{(1.24\cdot 10^{4})^2}{0.511\cdot 10^{6}}\,\frac{eV^2}{eV}$$

$$= 1.51\cdot 10^{2}\,eV$$

So, the electron has to run through a potential difference of 150 V.



$$d \sin\varphi = \lambda$$

$$\sin\varphi = \frac{10^{-10}\,m}{2.4\cdot 10^{-10}\,m} = 0.417$$

5. i) start from Bohr's model: energy of electron

$$E_e = \frac{1}{2} m_e v^2 - \frac{e^2}{r} = E_{kin} + E_{pot}$$

From $F_{centrifugal} = F_{coulomb}$, we know $m_e \frac{v^2}{r} = \frac{e^2}{r^2} = m_e a$

$$\overset{(3.11)}{\Longrightarrow} \quad E_e = -\frac{e^2}{2r} = \frac{1}{2} E_{pot} \overset{(3.12)}{=} -\frac{m_e e^4}{2L^2}$$

$$= -\frac{m_e e^4}{2\hbar^2 n^2} = -13.6\,eV \Big|_{n=1}$$

$$\Rightarrow \quad E_{pot} = -27.2\,eV \quad , \quad E_{kin} = 13.61\,eV$$

5 ii) Balmer: $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

frequency $f = \frac{c}{\lambda} = cR\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$

$$= 3 \cdot 10^8 \frac{m}{s} \cdot 1.097 \cdot 10^7 \frac{1}{m}\left(\frac{1}{9} - \frac{1}{16}\right)$$

$$= 1.56 \cdot 10^{14} \frac{1}{s}$$

frequency of revolution f_{revol}
calculate radius first: $-\frac{e^2}{2r} = -\frac{m_e e^4}{2\hbar^2 n^2}$

$$r_n = \frac{\hbar^2 n^2}{m_e e^2} = 0.529\,\mathring{A} \cdot n^2$$

velocity: $v_n = \sqrt{\frac{2E_{kin}}{m_e}} = \sqrt{\frac{2 \cdot 13.61\,eV}{n^2 m_e}} = \left(\frac{2 \cdot 13.61\,eV}{0.511 \cdot 10^6\,eV}\right)^{\frac{1}{2}} c \cdot \frac{1}{n}$

$$= 7.29 \cdot 10^{-3}\, c/n$$

$$f_{revol} = \frac{v_n}{2\pi r_n} =$$

$$= \frac{7.29 \cdot 10^{-3} \cdot 3 \cdot 10^8 \frac{m}{s} \cdot \frac{1}{n}}{2 \cdot \pi \cdot 0.529 \cdot 10^{-10}\,m \cdot n^2}$$

$$= 6.58 \cdot 10^{15} \frac{1}{s} \cdot \frac{1}{n^3} = \begin{cases} 2.4 \cdot 10^{14} \frac{1}{s} & \text{for } n=3 \\ 1.0 \cdot 10^{14} \frac{1}{s} & \text{for } n=4 \end{cases}$$

consistent with frequency derived above.

5iii) Lyman: $n_f = 1$

$$\frac{1}{\lambda} = R\left(\frac{1}{1} - \frac{1}{n_i^2}\right) = 1{,}097 \cdot 10^7 \frac{1}{m} \left(1 - \frac{1}{n_i^2}\right)$$

Longest wave length $n_i = 2$

$$\lambda_{long} = \frac{4}{3} \cdot (1{,}097 \cdot 10^7)^{-1} m =$$

shortest wavelength: $n_i \to \infty$

$$\lambda_{short} = (1{,}097 \cdot 10^7)^{-1} m =$$

6.) α is 2^+ charge, Cu has +-charge 29
potential energy is

$$k \frac{e^2}{r} \cdot 2 \cdot 29 = 13.9 MeV$$

$$k \frac{e^2}{r} = 8.988 \cdot 10^9 N \frac{m^2}{C^2} \cdot (1.6 \cdot 10^{-19} C)^2 \cdot \frac{1}{r}$$

$1 Nm = 1 J$
$= (1.6 \cdot 10^{-19}$

$$= \frac{m}{r} \cdot 8.988 \cdot 10^9 \cdot 1.6 \cdot 10^{-19} eV$$

$$= \frac{m}{r} \cdot 14.38 \cdot 10^{-10} eV \overset{!}{=} \frac{13.9 \cdot 10^6 eV}{2 \cdot 29}$$

$$r = \left(14.38 \cdot 10^{-10} eV \cdot \frac{1}{13.9 \cdot 10^6 eV} \cdot 2 \cdot 29\right) m = 60 \cdot 10^{-16} m$$
$$= \underline{\underline{6 \text{ fm}}}$$